The Cross-Section and Time-Series of Stock and Bond Returns

Ralph S.J. Koijen∗ Hanno Lustig† Stijn Van Nieuwerburgh‡
Chicago GSB UCLA and NBER NYU Stern and NBER

This version: February 2009

Abstract

The cross-section of returns of stock portfolios sorted along the book-to-market dimension can be understood with a one-factor model. The factor is the nominal bond risk premium, best measured as the Cochrane-Piazzesi (2005, CP) factor. This paper ties the pricing of stocks in the cross-section to the pricing of government bonds of various maturities, two literatures that have been developed largely in isolation. A parsimonious stochastic discount factor model can price both the cross-section of stock and bond returns. The mean average pricing error across 5 bond and 10 book-to-market stock portfolio returns is less than 60 basis points per year. The model also replicates the dynamics of bond yields as well as the time-series predictability of stock and bond returns. Its key feature is a non-zero risk price on the state variable that governs the bond risk premium. Value stocks are riskier because their returns are high when the bond risk premium is high. We trace back this risk to the fundamentals: CP predicts lower dividend growth on value stocks and higher dividend growth on growth stocks over the next 1-8 quarters. Since CP increases sharply over the course of a recession, it suggests that value stocks carry high returns because they have low cash-flow growth exactly when investors need it the most. We explore equilibrium asset pricing models’ ability to jointly account for the stock and bond returns patterns we document in the data.

JEL codes: G12.

Keywords: bond risk premium, cross-section of stock returns

∗ralph.koijen@chicagogsb.edu.
†hlustig@econ.ucla.edu.
‡svnieuwe@stern.nyu.edu.
The goal of this paper is to study a model that jointly accounts for stock and bond returns. According to the inter-temporal capital asset pricing model of Merton (1973), assets’ exposures to changing investment opportunities are important determinants of average returns, in addition to the market beta. Variables that capture changes in the investment opportunity set should be priced in the cross-section. For bonds, we know that the Cochrane and Piazzesi (2005) factor, henceforth \( CP \), is such a variable: it is the single best determinant of changes in future bond returns. A high \( CP \) signals high future bond returns. If bond return fluctuations are an important source of variation in the representative investor’s overall wealth, as Lustig, Van Nieuwerburgh, and Verdelhan (2008) argue, then the \( CP \) factor seems like the natural measure of investment opportunities in both bond and stock markets. Focusing on the cross-section of book-to-market and size-sorted stock portfolios, we find that high book-to-market (value) stocks are assets that have high returns when \( CP \) innovations are positive. Positive \( CP \) innovations signal improving investment opportunities: \( CP \) typically reaches its lowest value at the start of a recession and increases sharply as the recession draws to an end and good times are around the corner. Hence, value stocks are a poor hedge against deteriorating investment opportunities; this makes them command a high average return. Growth stocks do well when news comes in that future bond returns are lower, making them desirable for the investor. In equilibrium, they carry a low average return. We use this ICAPM insight as a starting point to build a no-arbitrage stochastic discount factor model that can account for the unconditional and conditional moments of stocks and bonds. The model takes as its point of departure the state-of-the-art term structure model of Cochrane and Piazzesi (2006). It adds a single factor, the aggregate stock market return, to arrive at a parsimonious model that achieves average pricing errors of 60 basis points per year on ten book-to-market sorted portfolios, the aggregate stock market return, and five maturity-sorted government bond portfolios.

In Section 1, we start by showing that the bond risk premium alone, proxied by the Cochrane-Piazzesi (henceforth CP) factor, explains at least 70% of the variation in monthly, quarterly, or annual stock returns on either the 25 size and value portfolios or the 10 decile book-to-market portfolios over the period 1952-2007. We find similar results when we proxy the bond risk premium by the slope of the yield curve. The returns of value stocks have higher exposure to the bond risk premium factor than the returns of growth stocks. This essentially amounts to a one-factor model of the cross-section of stock returns. This one-factor model explains as much of the cross-section of stock returns as the three-factor Fama-French or four-factor Carhart model. Next, we show that the cross-section of bond returns can be explained by a single yield (factor). Hence, both the cross-section of stocks and the cross-section of bonds have a one-factor structure, and in both cases, the relevant factor is a bond market factor.

This finding suggests a parsimonious unified model that can explain both the cross-section of
Section 2 develops such a model of the SDF. It starts from the state from the state-of-the-art in term structure modeling, by writing down the four factor Cochrane and Piazzesi (2005, 2006) model. The term structure is affine, and described by three factors (level, slope, and curvature) and the CP factor. The three factors explain 99% of the cross-sectional variation in yields, while the CP factor explains much of the variation in excess bond returns. To be precise, only the level factor is priced and its price of risk varies with the CP factor. This amounts to a parsimonious model with four state variables but only two non-zero price of risk parameters. We argue that if suffices to free up only one additional price of risk in this model in order to explain the cross-sectional variation in stock returns: the price of CP risk. Finally, we introduce a fifth factor, the stock market returns, which is necessary to get the level of the stock returns (or the equity premium) right. This amounts to a parsimonious SDF model with five factors, three non-zero prices of risk (level, CP, and market), and only four non-zero market price of risk parameters. This model generates mean absolute pricing errors below 60 basis points per year for ten decile book-to-market portfolios, the aggregate stock market portfolio, and five bond returns on bonds of various maturities. Finally, allowing the price of level risk to vary with the CP factor allows us to capture the dynamics of bond yields.

The CP factor displays interesting cyclical behavior. It is typically very low at the beginning of a recession (or a period of economic turmoil such as the summer of 1998), it increases during the recession, and peaks at the end of a recession. Hence, bond risk premia increase dramatically from a very low (often negative) value to a very high value over the course of a recession. The best time to enter into a value minus growth strategy is at the end of a recession, when CP is high. In contrast, the same value-growth strategy generates lower returns than the average value spread when entered when CP is low. The effects are economically large: when CP is in its highest quartile of observations, the value spread is 16.6%, while it is only 2.1% when CP is in the lowest quartile of its distribution.

We argue that this pattern can be understood in a rational model of stocks and bonds. In the absence of cash-flow risk, the higher loading of value portfolios’ returns on the bond risk premium (CP factor) are puzzling at first thought. One view, advocated by Lettau and Wachter (2007b), is that growth stocks pay dividends in the far future (long duration equity) and value firms in the near future (short duration equity). In such a world, the only difference between value and growth stocks is the timing of the cash-flows. For the same reason that long-maturity bonds are more sensitive to the bond risk premium than short maturity bonds, growth stocks would have a higher risk premium than value stocks, and a growth risk premium would arise. Hence, cash flows on value stocks must be riskier than those on growth stocks to offset this discount rate effect. One potential explanation for the higher cash-flow risk of value firms is that they have cash-flow growth that is more sensitive to the business cycle than that of growth firms: expected future dividend
growth on value stocks is relatively low at the start of a recession, when CP is low, and relatively high at the end of a recession, when CP is high.

This is exactly what we find in the equilibrium model of Section 3. It is a long-run risk model that provides a tractable laboratory to explore the connections between stock and bond returns. Three important changes to the benchmark Bansal and Yaron (2004) or Bansal and Shaliastovich (2007) model are necessary to account for the observed sensitivity of excess stock returns and dividend growth to the CP factor on value and growth portfolios. First, expected cash flow growth rates in the economy (x) must be positively correlated with economic uncertainty (σ2). Second, we need to allow for cross-sectional variation in expected dividend growth on different stock portfolios; not only by varying its loading with long-run risk (x), as in Bansal, Dittmar, and Lundblad (2005), but also its loading on economic uncertainty (σ2). Third, the economic uncertainty variable must be less persistent than commonly assumed.

The CP factor in the model has a correlation of one with economic uncertainty (σ2). It captures the state of the investment opportunity set, as it is the only variable that drives time series variation in any asset return; it is a perfect predictor of both future bond and stock returns. We estimate a lower persistence for σ2 than in the existing calibrations. This suggests that economic uncertainty behaves more like a recession variable than a low-frequency variable. Shocks to economic uncertainty carry a negative price of risk: marginal utility growth is high when uncertainty is high, so an investor is willing to pay a high price for assets with high cash-flows when uncertainty is high. In contrast, shocks to long-run risk (x) carry a positive price of risk because marginal utility growth is low when long-run growth is high. Because of the positive correlation between σ2 and xt, a high CPt not only implies high uncertainty but also high long-run growth. When economic uncertainty is less persistent than long-run risk, the effect of growth dominates that of uncertainty. Hence, a high CP corresponds to a period with low marginal utility growth. This explains the positive price of CP risk we estimated in the data. Assets that have high returns when CP is high are risky. Value stocks are such high CP-beta assets. The model shows that the underlying reason for the high risk premia on value stocks is that they have high expected future dividend growth exactly when marginal utility is low. This occurs at the end of a recession, when good times lie ahead.

Related Literature

The last twenty years have seen dramatic improvements in economists’ understanding of what determines differences in yields (Cox, Ingersoll, and Ross (1985), Duffie and Kan (1996), Dai and Singleton (2000, 2002)) and returns on bonds, as well as what determines heterogeneity in stock returns which differ by characteristics such as size and book-to-market value (Fama and French (1992)). Yet, these two literatures have developed largely separately and employ largely different
asset pricing factors. This is curious from the perspective of a complete markets model because both stock and bond prices equal the expected present discounted value of future cash-flows, discounted by the same stochastic discount factor. This paper contributes to both literatures and helps to bridge the gap.

On the theory side, representative-agent endowment models have been developed that are successful in accounting for many of the features of both stocks and bonds. Examples are the external habit model of Campbell and Cochrane (1999), whose implications for bonds were studied by Wachter (2006) and whose implications for the cross-section of stocks were studied separately by Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2006). Likewise, the implications of the long-run risk model of Bansal and Yaron (2004) for the term structure of interest rates were studied by Piazzesi and Schneider (2006), while Bansal, Dittmar, and Lundblad (2005) and Bansal, Dittmar, and Kiku (2007) separately study the implications for the cross-section of equity portfolios. In more recent work, Bansal and Shaliastovich (2007) study some of the joint properties of bond yields and stock returns, although not the ones we will focus on. Other recent work that ties stock and bond markets together in a general equilibrium model is Bekaert, Engstrom, and Grenadier (2005), Bekaert, Engstrom, and Xing (2008), Lettau and Wachter (2007a), Campbell, Sunderam, and Viceira (2008), and Gabaix (2008).

On the empirical side, the nominal short rate or the yield spread is routinely used either as a predictor of the aggregate return on the stock market or as a conditioning variable in an estimation of a conditional beta model of the cross-section of stock returns. Cochrane and Piazzesi (2005) show that a tent-shaped function of 5 forward rates forecasts not only nominal returns on bonds, but also has some forecasting ability for future aggregate stock market returns. Ang and Bekaert (2007) find some predictability of nominal short rates for future aggregate stock returns. Ferson and Harvey (1991) study stock and bond returns’ sensitivity to aggregate state variables, among which the slope of the yield curve. They conclude that time variation in equity risk premia is important for understanding the cross-sectional variation in size and industry equity portfolios and interest rate risk premia are important for understanding the cross-sectional variation in bond return portfolios. Fama and French (1993) find that three factors (market, size, and book-to-market) account for the variation in stock returns and two bond factors (excess return on long-term bond and a default spread) explain the variation in government and corporate bonds. They find that all of their stocks load in the same way on the term structure factors. Lustig, Van Nieuwerburgh, and Verdelhan (2008) price both nominal bond yields and the aggregate stock market return in a no-arbitrage model in order to measure the wealth-consumption ratio in the data; they do not study the cross-section of bond nor stock returns. Finally, Baker and Wurgler (2007) show that government bonds comove most strongly with “bond-like stocks”, which are stocks of large, mature, low-volatility, profitable, dividend-paying firms that are neither high growth nor distressed. They
propose that stocks and bonds are driven by a common sentiment indicator.

1 Empirical Analysis

Section 1.2 shows that the bond risk premium alone, either proxied by the Cochrane-Piazzesi (henceforth CP) factor or by the slope of the yield curve, explains a substantial fraction of the cross-sectional variation in monthly, quarterly, or annual stock returns over the period 1952-2007. Our main results are for the 10 decile book-to-market portfolios. We also report results for the 25 size and value portfolios, the 10 earnings-price portfolios and the 10 dividend-price portfolios. The main finding is that the returns of value stocks have higher exposure to the CP factor or to the yield spread than the returns of growth stocks. We compare this one-factor model of the cross-section of stock returns to the 3-factor Fama-French or 4-factor Carhart model. Sections 1.3 shows that the cross-section of bond returns can be explained by a single yield (factor). Hence, both the cross-section of stocks and the cross-section of bonds each have a one-factor structure, and in both cases the factor is a bond market variable. This suggests a unified model for pricing both stocks and bonds, which is what we provide in Section 3. But first, we briefly discuss the data and the construction of the proxies for the bond risk premium.

1.1 Data and the Cochrane-Piazzesi Factor

We use Fama-Bliss yield data for nominal bonds of maturities 1- through 5-years as well as the Fama risk-free rate on a three-month T-bill. These data are available for the sample June 1953-December 2007. We also use nominal stock returns from Kenneth French’ data library on 10 book-to-market sorted portfolios, 25 size and book-to-market portfolios, 10 earnings-price portfolios, and 10 dividend-price sorted portfolios. We use monthly, quarterly, and annual data.

Following the procedure outlined in Cochrane and Piazzesi (2005), we construct one- through five-year forward rates from our monthly, quarterly, or annual nominal yield data. From these yield series, we construct one-year excess holding period returns on two- through five-year nominal bonds, in excess of the one-year nominal yield. We regress the equally-weighted average of the one-year excess return on bonds of maturities of two, three, four, and five years on a constant, the one-year yield, and the two- through five-year forward rates. The regression coefficients display a tent-shaped function, very similar to the one reported in Cochrane and Piazzesi (2005). The variable \( CP_t \) is the fitted value of this regression. The regression \( R^2 \) is 24.1% with monthly, 29.3% with quarterly, and 37.7% with annual time-series. Figure 1 plots a time-series for the Cochrane-Piazzesi factor \( CP_t \); each panel corresponds to a different data frequency. However, the left-hand side variables are always one-year excess returns and the right-hand side variables annual forward rates.
Another, often-used proxy for the bond risk premium is the yield spread. We define it as the difference between the nominal yield on a five-year T-bond and the yield on a three-month T-bill. The $R^2$ of a regression of the equally-weighted average of the one-year excess return of bonds of maturities of two through five years on a constant and the yield spread gives an $R^2$ of 13.0% with monthly, 14.5% with quarterly, and 18.5% with annual time-series. The correlation between the yield spread and the $CP$ factor is 0.71 at monthly, 0.63 at quarterly, and also 0.63 at annual frequency. While the yield spread certainly has some predictive power for one-year ahead excess bond returns, it is a much weaker predictor than the $CP$ factor. Hence, we will exclusively focus on $CP$ from Section 2 onwards.

We also experimented with using both the long yield (5-year) and the short yield (3-month) as two separate predictors of the bond risk premium. Both are highly significant predictors of one-year excess bond returns; the long yield has a positive sign and the short yield a negative sign. The $R^2$ of the same bond return predictability regression is 18.4% at monthly, 20.0% at quarterly, and 24.0% at annual frequency. The combined predictive ability of the long and short yield for excess bond returns is substantially stronger than that of the yield spread, but still weaker than that of the $CP$ factor. The $CP$ factor is a parsimonious way to summarize information in excess bond returns.

### 1.2 Pricing the Cross-Section of Stocks with a Bond Factor

For each of the portfolio returns $i \in \{1, \cdots, N\}$ in the cross-section, we construct log excess returns corrected for a Jensen term: $r_{i,t+1}^{e} \equiv r_{i,t+1} - y_{t}^{b}(1) + 0.5\text{Var}[r_{i,t+1}]$, where the lowercase letters denote logs. We use log excess returns with Jensen adjustment for ease of comparison with the model in Section 2. All our results are quantitatively similar for gross excess returns and for gross returns. The returns are at monthly, at quarterly or at annual frequencies. We use a standard two-step estimation procedure; see Fama and MacBeth (1973) or Lettau and Ludvigson (2001). In the first step, we run $N$ time-series regressions of returns on the proxy for the bond risk premium, either the $CP$ factor or the yield spread, and a constant in order to estimate the factor beta. In the second step we run one cross-sectional regression of the $N$ average returns on the betas from the first step and a constant in order to estimate the market price of risk.

#### 1.2.1 First Stage Betas

Our benchmark set of test assets are 10 value-weighted decile portfolios sorted alongside the book-to-market dimension. The value spread, measured as $r_{10,t+1}^{e} - r_{1,t+1}^{e}$, is 5.26% per year. For comparison, the results are very similar if we define the yield spread as the difference between the 5-year yield and the 1-year T-note.
the value premium for gross returns is 5.32% per year. Figure 2 shows that value firms are exposed more heavily to \( CP \), our proxy for the bond risk premium. Each bar represents the slope coefficient (beta) of a time-series regression of a portfolio return on the \( CP \) factor. The data are monthly (1952.6-2007.12) in the top panel, quarterly in the middle panel (1952.II-2007.IV), and annual in the bottom panel (1953-2007). The book-to-market ratio increases reading from the left to the right. There is a clear pattern in these betas: they are much larger for value than for growth firms. At monthly and annual frequency, the CP-beta for extreme growth is negative whereas it is high and positive at every frequency for the extreme value portfolio.

[Figure 2 about here.]

The same pattern of exposures arises when we add a size dimension to the cross-section. Figure 3 shows the contemporaneous \( CP \) betas for the 25 market equity- and book-to-market equity-sorted portfolios (5-by-5 sort). The first five bars are for the smallest quintile of firms; the last five bars are for the biggest quintile of firms. Within each group of five, the book-to-market ratio increases. There is the same clear pattern in these betas: they are much larger for value than for growth firms. While the difference in exposure between the highest and lowest book-to-market group is more pronounced for small firms than for big firms, there is less variation along the size dimension.

[Figure 3 about here.]

The same pattern in betas also arises when we use the contemporaneous yield spread as an alternative proxy for the bond risk premium. For both the 10 book-to-market portfolios and the 25 size and value portfolios, we find that value stocks have a high positive loading on the yield spread, whereas growth stocks have a much lower exposure. The difference is larger for small stocks. We summarize these findings in Figure 4 which shows betas for these two sets of test assets at all three frequencies.

[Figure 4 about here.]

1.2.2 Second Stage Regression

In a second stage, we run a cross-sectional regression of average excess stock returns on the first-stage beta and a constant in order to estimate the market price of risk \( \tilde{\Lambda} \). Table II shows the results; the top panel is for the 10 BM portfolios, the bottom panel for the 25 size and BM portfolios. The left panel uses \( CP \) as a proxy for the bond risk premium, the right panel uses the yield spread instead. In each of the four panels, we report results at monthly (M), quarterly (Q), and annual (A) frequency. Point estimates for the price of risk \( \tilde{\Lambda} \) are directly comparable across frequencies. For
ease of comparison, the second-stage intercept $\alpha$ is expressed as a percent per year by multiplying
the monthly intercept by 12 and the quarterly intercept by 4.

Using $CP$, we find a highly significant market price of risk estimate between 1.6 (M) and 3
(A) for the 10 BM portfolios and between 2.2 (M) and 3.3 (A) for the 25 size and BM portfolios.
The second-stage regression has a high $R^2$: around 75% for the 10 and around 80% for the 25
portfolios. The pricing errors are small: the mean absolute pricing error (MAPE) across the 10
portfolios is between 65 basis points per year (M) and 75 basis points (A) for the 10 BM portfolios,
the root mean squared error (RMSE) between 75 (M) and 93 (A) basis points per year. For the
25 portfolios, the MAPE is between 80 and 109 and the RMSE between 104 and 144 basis points
per year. Figure 5 plots average realized excess returns against predicted excess returns, formed
as $\alpha + \beta \lambda$, where $\alpha$ is the second-stage regression intercept. The left row is for the 10 BM, the
right row for the 25FF portfolios. The sampling frequency is monthly in the top panels, quarterly
in the middle panels, and annual in the bottom panels. It confirms visually that a large fraction
of the cross-sectional variation in stock returns is explained by their exposure to the bond risk
premium. Combining results from the two stages, value firms (BM10) and growth firms (BM1)
have a differential exposure to $CP$ of 3.4 in the monthly data. With a market price of risk of 1.57, this translates into a value premium of 5.3%, which is essentially the entire value premium
observed in the data.

[Table 1 about here.]

[Figure 5 about here.]

We repeat the analysis with the yield spread, in the right panel of Table 1. There too, we find
significantly positive market prices of risk. The pricing errors are somewhat larger at monthly and
quarterly frequencies, but somewhat smaller at annual frequency. The regression $R^2$ vary between
55% and 83%. Because exposure to the yield spread carries a positive risk price, the differential
exposure of value and growth stocks leads to a value spread on value than on growth stocks.
One advantage of the yield spread is that the second stage regression no longer has a significant
intercept $\alpha$ in the monthly data. The positive $\alpha$’s in all other specifications, suggest that not all
cross-sectional variation is explained by the yield spread or the $CP$ factor. In particular, there
seems to be a common factor missing that increases the returns on all portfolios. We will introduce
such a “market factor” in Section 2. Nevertheless, the small pricing errors throughout show that
a single bond factor, linked to the bond risk premium, can explain most of the variation in the
cross-section of stock returns around a common market return. We consider the regression analysis
of this section as preliminary evidence. Section 2 explores the links between stock returns, bond
returns, and bond yields in much more detail.
1.2.3 Sub-sample Analysis

As a first robustness check, we repeat the analysis with returns instead of excess returns. The results are very similar and are available upon request. As a second robustness check, we conduct a sub-sample analysis. We present two sets of results in Table 2 for the 10 BM portfolios (top panel) and the 25 size and BM portfolios at monthly frequency (bottom panel). The first set of results uses the \( CP \) factor constructed from the full sample (left panel); the second set of results re-estimates the \( CP \) factor over the sub-sample in question (right panel). If we start the analysis in 1963, an often used starting point for cross-sectional equity analysis (e.g., Fama and French (1993)), we find somewhat stronger results. The pattern in the betas is similar as in the full sample, but the market price of risk estimate increases and so does the second-stage \( R^2 \). The mean absolute pricing errors drop another 4 basis points to 60 basis points per year. We also split the full sample into two sub-samples: 1952-1985 and 1986-2007. The latter period corresponds to the period after the high-inflation era, and has lower and declining bond risk premia (see Figure 1). The results remain strong in both sub-samples: pricing errors generally remain below 1.5% per year and the cross-sectional \( R^2 \)s remain high. The market price of risk estimates are lower, especially in the last sub-sample, but they remain significantly positive in every instance. Because \( CP \) is trending down in this sub-sample, all \( CP \) betas are negative, and they are much more negative for value than for growth stocks. Given the positive price of risk (and the large intercept), that spread in exposures still explains the (positive) value premium.

[Table 2 about here.]

1.2.4 Comparison with Three-Factor Model

As a last robustness exercise, we compare our results to the celebrated Fama-French three-factor and Carhart four-factor models. For brevity, we focus on the 25 size and BM portfolios and on monthly data. The results for quarterly and annual data are similar. The factors are the log excess return on the market including a Jensen adjustment (6.61% per year on average), the log return on SMB (1.57% per year on average), the log return on the HML (4.42% per year on average), and the log return on the UMD portfolio (9.44% per year on average). The first two columns of Table 3 repeat the results for the models with only \( CP \) or the yield spread as pricing factor. Columns 3 and 4 present the results from the standard three-factor (3-f) and four-factor model (4-f). The main observation is that the pricing errors are similar to, and if anything larger than, the pricing errors for our one-factor model with \( CP \) as factor, reported in Column 1. Additionally, the three-factor model in Column 3 has a negative price of market risk, inconsistent with theory, and the cross-sectional intercept is very large. The latter two issues are mitigated by including the momentum factor in Column 4.
Does the bond risk premium survive the inclusion of the three- or four-factor model’s risk factors? Columns 5 through 8 show that both the \( CP \) factor and the yield spread remain significantly priced risk factors once the market, SMB, HML, and UMD are added. Adding the \( CP \) factor to the 3-factor model lowers the MAPE from .91% per year to .67% per year and increases the cross-sectional \( R^2 \) from 75.9% (Column 3) to 85.8% (Column 5), amounting to a substantial improvement in fit. Adding the \( CP \) factor to the 4-factor model lowers the MAPE from .86% to .69% and increases the \( R^2 \) from 81.0% (Column 4) to 87.0% (Column 7). Adding the yield spread leads to even bigger gains in \( R^2 \) and bigger reductions in pricing errors. The pricing errors of the resulting 4-factor and 5-factor models in Columns 6 and 8 are .52% and .47% per year, half the size of the model without the yield spread factor. The \( R^2 \)s are 93.6% and 94.4%.

Interestingly, the addition of the \( CP \) factor or the yield spread increases the explanatory power of the SMB factor and of the HML factor. While the point estimates are somewhat smaller, the t-stat on the SMB factor doubles and that on HML increases by 50% in many specifications. Including the bond risk premium seems to crystalize the effects of the size and value risk factors. In contrast, the momentum factor loses its significance for explaining the size and value portfolios.

[Table 3 about here.]

1.2.5 Non-Linearity

The bond risk premium proxies are clearly related to the state of the economy. Figure 6 plots the \( CP \) factor (top panel) and the yield spread (bottom panel) against the NBER recession dates. Both variables tend to be low at the beginning of a recession, increase substantially during a recession, and tend to peak at the end of a recession. During the average recession in the sample, \( CP \) increases by 224 basis points (1.27 standard deviations), typically from a negative to a positive value. Likewise, the yield spread increases by 144 basis points (1.40 standard deviations) from a negative to a positive value. The fact that the \( CP \) factor and the yield spread peak at the end of a recession (the through of the business cycle), when good economic times are around the corner and marginal utility growth for the representative investor is presumably low, can explain the positive price of \( CP \) or yield spread risk we found above.

[Figure 6 about here.]

Returns on value and growth stocks are also linked to the cycle. An investment strategy which is long value and short growth stocks has low returns when entered at the start of a recession and high returns when entered at the end of a recession. Average monthly holding period returns on the value-minus-growth strategy are only 1.5% during NBER recessions (15% of the months in the sample) but 5.8% the rest of the time. So, the fortunes of a value-growth strategy reverse over the course of a recession.
Not all periods with a high (low) CP factor or yield spread observations occur at the end (beginning) of recessions. However, several of these low-to-high swings in the CP factor are also associated with times of economic uncertainty, such as 1987 (stock market crash), 1998 (Asian and Russian crisis, LCTM), or 2003 (jobless recovery). An even starker difference between the returns on value and growth stock emerges when we condition on the top quartile of the CP observations or the top quartile of yield spread observations. The CP factor cutoffs are -0.45% (25\text{th} percentile), .58% (median), and 1.60% (75\text{th} percentile), while the yield spread cutoffs are 0.28% (25\text{th} percentile), 0.94% (median), and 1.68% (75\text{th} percentile). Figure 7 shows that the value spread is highest when CP is high (first row, left panel) or when the yield spread is high (first row, right panel). In that quartile, the value spread is 16.9% for CP and 14.5% for the yield spread. As the other rows show, there is little spread in the other quartiles. The value spread is almost exclusively concentrated in periods of high CP or yield spread realizations.

This non-linear contemporaneous relationship between the value spread and the CP factor naturally suggests a refinement of the cross-sectional stock return regression we ran above. Instead of using \(CP_{t+1}\) as an explanatory variable, it suggests using \(CP_{t+1}\mathbb{1}_{CP_{t+1}>0.016}\), where \(\mathbb{1}\) denotes an indicator variable and .016 is the cutoff for the fourth quartile of CP. In such a specification, innovations to CP are only priced when CP is high. This occurs typically at the end of a recession, when good times are around the corner. Using the ten book-to-market sorted portfolios, the resulting market price of risk parameter in the second-stage Fama-McBeth regression is 1.01 with a t-stat of 6.25. The pricing errors are 62 (MAPE) and 66 (RMSE) basis points and the cross-sectional \(R^2\) is 82.98%. This model further reduces pricing errors by 3 (MAPE) and 11 (RMSE) basis points and increases the \(R^2\) by 5 percentage points over the linear specification. A similar improvement occurs if we use the same non-linear function of the yield spread, \(yspr_{t+1}\mathbb{1}_{yspr_{t+1}>0.0168}\). The MAPE goes down by 20 and the RMSE by 12 basis points, while the cross-sectional \(R^2\) improves by 9.3 percentage points.

1.3 Pricing the Cross-Section of Bonds with a Bond Factor

While the CP factor is both a strong predictor of future bond returns and a strong explanatory variable of differences between average returns of value and growth stocks, it is well known that it is not a good explanatory variable of differences between average returns on long- and short-term bonds. We now show that another bond factor, the level of the term structure, has significant ability to explain the cross-section of bond returns. As a first measure of the level factor, we use the five-year Fama-Bliss bond yield. As a second measure, we use the first principal component of yields as the level factor, extracted from the cross-section of 1- through 5-year Fama-Bliss yields.
We use CRSP excess bond returns on portfolios of maturities 1, 3, 5, 7, and 10 years at monthly, quarterly, or annual frequency. Average excess returns are respectively 1.08%, 1.24%, 1.53%, 1.75%, and 1.38% per year. The sample is again June 1952 until December 2007.

Figure 8 shows the level betas of excess bond returns on the 5 maturity-sorted bond portfolios. They are decreasing from short to long maturities and generally negative. Table 4 shows the corresponding market prices of risk using $y^8(20)$ (on the left) and using the first principal component (on the right). The market price of risk is estimated to be negative and significant, as predicted by theory. Intuitively, investors are willing to pay for assets with high returns when yields increase, which are times in which wealth decreases and marginal utility increases. Long bonds have more negative level betas: their returns fall by more when yields increase and hence they earn higher average returns. The pricing errors on the 5 bond portfolios are small; the MAPE is 8 basis points and the RMSE 11 basis points per year. The explanatory power of the 20-quarter yield and the standard level factor are similar. The cross-sectional $R^2$ is around 75%. The negative betas combined with the negative market price of risk allow this one-factor model to capture 0.6% of the observed 0.8% spread between the excess return on the 10-year and the 1-year bonds.

Combined with the previous section, these results suggest that two term structure variables, a proxy for the bond risk premium $CP$ and a level factor, can help explain both the cross-section of stock and bond returns. Section 2 will propose a model that can fit both. In addition to matching cross-sectional variation, it will also seek to match the levels of the risk premia on stocks and bonds. Furthermore, it will do so for a single market price of risk vector $\tilde{\Lambda}$, not a different one for stocks and bonds.

1.4 Predictability

The previous analysis was concerned with understanding the cross-section of average stock and bond returns. However, the covariance of $CP_{t+1}$ or $yspr_{t+1}$ with excess returns $r^e_{t+1}$ is the weighted average of two components: a covariance of $r^e_{t+1}$ with innovations in the bond risk premium variable and a covariance of $r^e_{t+1}$ with the lagged bond risk premium. In Section 2.5 we make that link explicit. The covariance of excess returns with the lagged $CP$ factor (lagged yield spread) measures the predictability of excess returns by $CP$ ($yspr$). From Cochrane and Piazzesi (2005), we know that lagged $CP$ predicts future bond returns of various maturities and at different holding periods. There is also abundant evidence that the yield spread forecasts future stock market returns; e.g. Ang, Piazzesi, and Wei (2006). This raises the question of whether $CP$, which is a good proxy for expected bond returns, is also a good proxy for expected stock returns.
We investigate whether \( CP \) forecasts (i) future aggregate stock returns, and (ii) the spread between future value and future growth returns. To that end, we use monthly data and form cumulative, discounted holding period returns of horizons of 3 months up to 5 years\(^2\). The first 59 months are discarded so that all cumulative returns are computed over the same sample. All cumulative results are expressed on a per annum basis so that the slope coefficients have comparable magnitudes across holding periods. The top panel of Table 5 shows results from regressing the cumulative value-growth returns on a constant and the lagged \( CP \) factor. The value-minus-growth portfolio return is predictable by \( CP \), but only significantly so at the 2 year horizon based on Newey-West t-stats. Throughout, the \( R^2 \) statistics are low. The result can be traced back to the value stocks (portfolio 10) whose returns are somewhat predictable by \( CP \). In the left columns of the bottom panel, we find no evidence for predictability of future excess stock market returns on the value-weighted CRSP portfolio by lagged \( CP \). The effect is even weaker than for the value minus growth portfolio. For completeness, the right panel uses the yield spread as predictor instead of the \( CP \) factor. The yield spread also has positive forecasting power for value minus growth returns at horizons of 3-quarters to 2 years, but its effect is not statistically different from zero and the \( R^2 \) is low. The effect of the yield spread is stronger than that of \( CP \) when it comes to forecasting the aggregate stock market return.

[Table 5 about here.]

However, the results of Section 1.2.5 suggest that there is only a value premium in 25\% of the months in the sample. This raises the question of whether the predictability of the value minus growth return may also be non-linear. We indeed find strong evidence for predictability of the \( CP \) factor for future value minus growth returns, but only in months in which \( CP \) is in its highest quartile. Table 6 shows results from regressing the cumulative value-growth returns on a constant and a non-linear function of the \( CP \) factor, \( CP_{t \geq CP > 0.016} \), the lagged version of the variable that did such a good job explaining the cross-section of book-to-market portfolio returns. The top left panel shows a positive coefficient which is statistically significant at horizons of 1 quarter up to 4 years. The \( R^2 \) are respectable, peaking at almost 7\% at the 2-year horizon. This non-linear function also shows improved predictability for the aggregate stock market return at horizons of 4 and 5 years. However, the result is much weaker than for value minus growth. For completeness, we repeat the exercise with a similar non-linear function of the yield spread (right panels). We find similar improvements in the predictability of value minus growth. However, the non-linear function of the yield spread no longer predicts the aggregate stock market return at horizons below 4 years. The main conclusion is that the same variable that explains the cross-section of book-to-market portfolio returns also has time-series predictability for the high minus low book-to-market returns.

---

\(^2\)The discounting of future returns on each asset \( i \) is done by the constant \((\kappa_i)^{-1} = \exp(A_{0i})/(1 + \exp(A_{0i}))\), where \( A_{0i} \) is the mean log price-dividend ratio on portfolio \( i \).
portfolio. The stochastic discount factor model of Section 2 will match both the unconditional moments of excess returns and the predictability of the value-minus-growth strategy by the CP factor discussed here.

[Table 6 about here.]

2 A Unified Model of Stock and Bond Returns

The previous analysis suggests that the level, the cross-section, and the time-variation of expected stock returns are driven by the market factor and the CP factor, while the level, the cross-section, and the time-variation of expected bond returns are driven by the level factor and the CP factor. In this section, we develop a parsimonious stochastic-discount factor (SDF) model that captures all of these features. We show that this model is also able to match the time-series and cross-sectional properties of bond yields, thereby providing a unified pricing framework for stocks and bonds. For brevity, we focus on the 10 book-to-market sorted stock portfolios and study returns at the monthly frequency. Section 3 then develops a general equilibrium model that starts from preferences and from dividend growth rates on the stock portfolios. There, returns are generated inside the model, while here, they are taken as given.

2.1 Setup

We model the monthly dynamics of the demeaned state $X$ as:

$$X_{t+1} = \Gamma X_t + \varepsilon^X_{t+1},$$  \hspace{1cm} (1)

with Gaussian innovations $\varepsilon^X_{t+1} \sim \mathcal{N}(0, \Sigma_X)$. The state vector contains five variables: the CP factor, the level factor, the slope factor, the curvature factor, and aggregate stock market returns. To construct the level, slope, and curvature factors, we regress Fama-Bliss forward rates on bonds of maturities 1-5 years (the same rates that were used in the construction of CP) on the CP factor. Level, slope, and curvature are the first three principal components of the residuals of this regression, and therefore orthogonal to CP. The first four factors have been suggested by Cochrane and Piazzesi (2006) to price the nominal term structure of interest rates. Even though we use five factors, only three of them will carry a non-zero price of risk in our preferred specification. The other factors are therefore relevant only to form expectations of the future state using (1).

The stochastic discount factor is modeled as:

$$M_{t+1} = \exp\left(-y^S_t(1) - \frac{1}{2} \Lambda_t\Sigma_X\Lambda_t - \Lambda_t\varepsilon^X_{t+1}\right),$$  \hspace{1cm} (2)
with \( y_t^S(1) \) the 1-month short rate and market prices of risk that are affine in the state \( \Lambda_t = \Lambda_0 + \Lambda_1 X_t \). Geometric excess returns (including Jensen adjustment) on all assets are modeled as:

\[
r_{t+1} = r_t + 1 \frac{1}{2} \text{var}_t [r_{t+1}] = \delta_0 + \delta_1 CP_t + \varepsilon_{t+1}^R,
\]

with Gaussian innovations \( \varepsilon_{t+1}^R \sim \mathcal{N}(0, \Sigma_R) \) and covariance matrix \( \text{cov} (\varepsilon_{t+1}^R, \varepsilon_{t+1}'X_t) = \Sigma_{XR} \). We focus on the ability of the CP to predict returns to replicate the predictability results in Section 1.4.

The no-arbitrage condition for each asset is given by:

\[
\log E_t [M_{t+1} R_{t+1}] = 0,
\]

which implies:

\[
E_t [r_{t+1}^e] = -\text{cov}_t (r_{t+1}^e, m_{t+1}) = \text{cov}_t (\varepsilon_{t+1}^R, \varepsilon_{t+1}'X_t) \Lambda_t = \Sigma_{XR} (\Lambda_0 + \Lambda_1 X_t).
\]

This shows that \( \delta_0 = \Sigma_{XR} \Lambda_0 \) and \( \delta_1 = \Sigma_{XR} \Lambda_1 \). Unconditional expected excess returns are computed by taking the unconditional expectation of (4):

\[
E [r_{t+1}^e] = \Sigma_{XR} (\Lambda_0 + \Lambda_1 E [X_t]).
\]

This section proceeds as follows. We first show that a parsimonious model is able to price the cross-section of unconditional expected stock and bond returns. Next, we demonstrate that the model is able to price the time series and cross-section of bond yields, and that it can match the predictability of the aggregate stock market and the return on a long growth-short value strategy. The latter are conditional expected stock return moments. Finally, we connect the results from this section to those of the previous section.

Our test assets are the 10 portfolios sorted on their book-to-market ratio, the value-weighted market return from CRSP, and bond returns with maturities 1, 2, 5, 7, and 10 years from CRSP. Our estimation procedure estimates the dynamics of the state by OLS in a first step. It then finds the market price of risk coefficients \( \Lambda_0 \) that minimize the pricing errors on the test assets in a second step. All 16 moments are weighted equally. In a third step, we estimate the parameters in \( \Lambda_1 \) to fit the time series and cross-section of yields as well as the return predictability moments.

---

\(^3\)We will assume that the Jensen adjustment \( \frac{1}{2} \text{var}_t (r_{t+1}) \) is a constant. This would arise under homoscedasticity assumptions on return innovations. This adjustment is tiny in practise.

\(^4\)It is straightforward to extend our results and to include additional predictors of either stock or bond returns.
2.2 The Cross-section of Unconditional Expected Returns

In this section, we try to match the cross-section of unconditional, i.e. average, expected stock and bond returns. We switch off all time variation in risk prices ($\Lambda_1 = 0$) because, at this stage, we are only interested in unconditional pricing errors. We optimize over (a subset of) $\Lambda_0$. To fix ideas, we start by pricing the 16 securities with a risk-neutral pricing kernel $M_{t+1} = \exp \{ -y_t^S(1) \}$. That is, all prices of risk are zero: $\Lambda_0 = 0$. The first column of Table 7, labeled SDF1, thus recovers the risk premia we want to explain. The value spread (BM10-BM1) is 5.3% per year. Bond risk premia are not monotone in maturity and smaller than equity risk premia. The MAPE across all securities equals 6.1%.

Our first candidate SDF to explain the stock and bond risk premia is the one proposed by Cochrane and Piazzesi (2006). This is a natural candidate because their model does a great job pricing the nominal term structure of bond yields. Following Cochrane and Piazzesi (2006), we only allow the market price of level risk to be non-zero. This implies one non-zero element in $\Lambda_0$. The second column of Table 7 shows that the best-fitting SDF, SDF2, is unable to jointly explain the cross-section of stock and bond returns. The MAPE is 3.96%. All pricing errors on the stock portfolios are large and positive, while all pricing errors on the bond portfolios are large and negative. The reason that this bond pricing model does not do better for the bond return portfolios is that the stock return risk premia command most attention because they are so much larger in magnitude. Consequently, the estimation concentrates its efforts on reducing the pricing errors of stocks. To illustrate that this bond kernel is able to price the cross-section of bonds, we estimate the same model by minimizing only the bond pricing errors. The third column of Table 7 illustrates that the bond pricing errors are small in this case, but this pricing kernel SDF2 is not able to price the cross-section of expected stock returns nor the expected return on the aggregate stock market. The MAPE increases to 5.06%.

Our second candidate SDF to explain stock and bond risk premia is the canonical equity pricing model, the Capital Asset Pricing Model. The fourth column of Table 7 reports pricing errors for the CAPM. The only non-zero price of risk is the one corresponding to the aggregate stock market return. This SDF3 model is again unable to jointly price stock and bond returns. The MAPE is 1.54%. One valuable feature is that pricing errors of book-to-market portfolio returns go through zero. This means that the model gets the level of all expected stock returns right. However, their pattern clearly shows the value spread. Pricing errors on bond portfolios are sizeable as well and are all positive.

Having concluded that both the bond SDF and the stock SDF offer crucial ingredients to price bond returns and stock returns, but that neither is able to satisfactorily price both cross-sections, we now turn to our unified model. We allow the price of risks of the $CP$ factor, the level factor, and the aggregate market return to be non-zero. For pricing purposes, this is a three-factor model.
The three elements in $\Lambda_0$ are estimated to minimize the 16 pricing errors. As before, there is no time variation in the prices of risk: $\Lambda_1 = 0$. We call this model SDF4; the fifth column shows the results. The MAPE falls to a low 61bp. This is essentially the same magnitude pricing error we found in Section [1] for the 10 stock portfolios. Using one kernel with three price of risk parameters only, we are able to match the level and the pattern in stock and bond risk premia jointly.

The remaining issue is that there remains somewhat of a predictable pattern in bond risk premia and that the bond risk premia are still relatively large. The level factor is the risk factor that is responsible for pricing these bond portfolios. The problem is that the covariance of the innovations to the level factor and the innovations to bond returns of various maturities goes through zero. That implies that the level factor can only “tilt” the pattern of risk premia, but cannot scale all risk premia simultaneously.

Based on this insight, we propose a modified level factor. This modified level factor is constructed as a weighted average of the Fama-Bliss yields of maturities one through five years, just like the standard level factor. However, the weights are additional parameters chosen to further minimize pricing errors under the restriction that they sum to one. The discipline comes from the fact that the resulting level factor is priced and needs to explain the cross-section of expected stock and bond returns. As discussed below, further discipline comes from pricing the cross-section of nominal bond yields. With the modified level factor in hand, we construct a corresponding modified slope and curvature factors as follows. We project forward rates on the (original) $CP$ factor and the modified level factor and collect the residuals. The modified slope and modified curvature factors are the first two principal components of these residuals. The modified level factor has a correlation of 80% with the standard level factor. The slope and the modified slope factor have a correlation of 64%, while the curvature and the modified curvature factor have a correlation of 67%.

Our final model, SDF5, contains the CP factor, the market factor, and the modified level, slope, and curvature factors as its state vector. As for SDF4, we only estimate 3 non-zero market prices of risk. This model’s pricing errors are reported in the sixth column of Table [7]. The MAPE is only 51 basis points. There hardly is a value spread in the pricing errors: The spread between the extreme portfolios is about 1% per year. Furthermore, the bond pricing errors are much lower than for SDF4, between 19 and 32 basis points. There are no more pronounced patterns in the pricing errors of either bonds or stocks. Finally, the price of CP risk and the price of market risk are estimated to be positive, while the price of (modified) level factor risk is negative. These are the signs predicted by theory.

[Table 7 about here.]
2.3 Implications for the Yield Curve

The previous section showed that a parsimonious SDF model was able to price unconditional excess returns on both stocks and bonds. An important question is how well the same SDF model does in pricing the term structure of nominal bond yields. After all, any SDF model has implications for both (bond) returns and (bond) yields. It is well known that three factors (level, slope, and curvature) are necessary to adequately describe bond yield dynamics, e.g., Dai and Singleton (2000, 2002). While the slope and curvature factor were unnecessary to price average returns, as shown above, we included them in the state vector $X$ because they are necessary to adequately describe yields. They enter through the expectations formation in the state variable dynamics in equation (1). Matching yield dynamics requires only non-zero elements in $\Lambda$; namely $\Lambda^{(1)}(2,1)$. This is the insight of (Cochrane and Piazzesi 2006), that allowing the market price of level risk to move with the CP factor suffices to match the dynamics of bond yields.

Our SDF model implies an essentially affine term structure of yields. We model the nominal short rate as

$$y_t^s(1) = \xi_0 + \xi_1^t X_t.$$  

Then, the SDF implies that the price of a nominal bond of maturity $\tau$ is exponentially affine in the state variables $X$:

$$P_t^s(\tau) = \exp\left\{ A^s(\tau) + B^s(\tau)X_t \right\}.$$  

By no-arbitrage, we have:

$$P_t^s(\tau) = E_t \left[ M_{t+1} P_{t+1}^s(\tau - 1) \right]$$

$$= \exp \left\{ -\xi_0 - \xi_1^t X_t + A^s(\tau - 1) + B^s(\tau - 1)\mu_X + B^s(\tau - 1)\Gamma X_t - \Lambda_0^t \Sigma B^s(\tau - 1) + \frac{1}{2} B^s(\tau - 1)\Sigma X B^s(\tau - 1) \right\},$$

which implies that $A^s(\tau)$ and $B^s(\tau)$ follow from the recursions:

$$A^s(\tau) = -\xi_0 + A^s(\tau - 1) + B^s(\tau - 1)\mu_X - \Lambda_0^t \Sigma X B^s(\tau - 1) + \frac{1}{2} B^s(\tau - 1)\Sigma X B^s(\tau - 1),$$

$$B^s(\tau) = -\xi_1^t + B^s(\tau - 1)\Gamma - B^s(\tau - 1)\Sigma X \Lambda_1,$$

with initial conditions $A^s(0) = 0$ and $B^s(0) = 0_{5 \times 1}$.

We estimate the parameters in three steps. The first two steps are the same as in the unconditional analysis described above. The third step estimates $\xi_0$, $\xi_1$, and the first non-zero entry of $\Lambda_1$, $\Lambda_1(1,1)$, to match nominal bond yields. In particular, we minimize the distance between the five annual yields from the Fama-Bliss data set (maturities 1 through 5 years) and the corresponding...
yields implied by the SDF:

\[
\min_{\xi_0, \xi_1, \Lambda_{1(2,1)}} \left\{ \sum_{n=1}^{5} \sum_{t=1}^{T} \left( y_t^S (12 \times n) + \frac{A^S (12 \times n)}{12 \times n} + \frac{1}{12 \times n} B^S (12 \times n) X_t \right)^2 \right\},
\]

where the factor “12” is introduced because we have a monthly model and we price annual yields. In optimizing over \( \Lambda_1 \), we modify \( \Lambda_0 \) to \( \Lambda_0 \equiv \Lambda_0 - \Lambda_1 E[X_t] \) to ensure that the unconditional asset pricing results from the previous section are unaffected. In related work, Adrian and Moench (2008) match bond returns first and then study the implications for yields.

The model SDF5 does a very nice job matching yields. The annualized standard deviation of the pricing errors of yields equal 18bp, 10bp, 12bp, 21bp, and 25bp for 1-5 year yields. Figure 9 plots this model’s implications for the yields of maturities one through five years. The parameter \( \Lambda_{1(2,1)} \) is estimated to be negative (\( \hat{\Lambda}_{1(2,1)} = -2160.2 \)), consistent with the results in Cochrane and Piazzesi (2006). A negative sign means that an increase in \( CP \) leads to higher bond risk premia because the price of level risk is negative and a higher \( CP \) makes it more negative.

[Figure 9 about here.]

2.4 Return Predictability

We study conditional stock return moments in this section. Our aim is to match the return predictability facts documented in Section 1.4: the predictability of aggregate stock market returns and returns on the value-minus-growth strategy by the \( CP \) factor. Matching these two moments exactly requires two more non-zero elements in \( \Lambda_1 \), \( \Lambda_{1(1,1)} \), and \( \Lambda_{1(5,1)} \), which capture the time variation in the market price of \( CP \) risk and the market price of market risk generated by the \( CP \) factor. These moments can be written as

\[
(\sigma_{XR}^{BM10} - \sigma_{XR}^{BM1}) \Lambda_1 = \delta_1^{BM10-BM1} \quad \text{and} \quad \sigma_{XR}^{Mkt} \Lambda_1 = \delta_1^{Mkt},
\]

where \( \delta_1^j \) denotes the predictive coefficient form equation (3). We find that the price of \( CP \) risk and aggregate stock market risk both need to be increasing in the \( CP \) factor (\( \hat{\Lambda}_{1(1,1)} = 2739.4 \) and \( \hat{\Lambda}_{1(5,1)} = 38.0 \)), consistent with the results in Section 1.4. Taken together with the results from the previous section, the SDF5 model is able to not only match unconditional excess returns in the cross-section of stocks and bonds, but also bond yields, and conditional stock return moments. It does so with three priced risk factors, each of whom varies over time only with the \( CP \) factor.

2.5 Connection with Cross-sectional Regressions

The last missing piece is to tie the results of this section back to the regression analysis of Section 1. There, we estimated average returns as the product of an unconditional \( CP \)-beta and a price of
CP risk $\tilde{\lambda}$

$$E[r_{t+1}^e] = \beta \tilde{\lambda} = \text{Cov}[r_{t+1}^e, CP_{t+1}] Var[CP_{t+1}]^{-1} Var[CP_{t+1}] E[\Lambda_t],$$

where $\Lambda_t$ refers to the market price of risk notation of this section. Because expected excess returns only depend on $CP_t$ according to equation (3), we can write the “contemporaneous CP beta” from Section 1 as the weighted average of the lagged betas and the innovation betas with weights $w_1$ and $w_2$:

$$\beta = \text{Cov}[r_{t+1}^e, CP_{t+1}] Var[CP_{t+1}]^{-1} = w_1 \beta^{\text{Lagged}} + w_2 \beta^{\text{Innov}},$$

$$w_1 = \text{Cov}[X_t, CP_t] \Gamma'_{CP} Var[CP_{t+1}]^{-1},$$

$$w_2 = Var[\varepsilon_{t+1}^{CP}] Var[CP_{t+1}]^{-1},$$

where $\Gamma_{CP}$ selects the first row of the companion matrix $\Gamma$, the lagged CP beta is $\delta_1$, and the innovation beta is defined by the regression:

$$\varepsilon^R_{t+1} = \beta^{\text{Innov}} \varepsilon_{t+1}^{CP} + \varepsilon^R_{t+1}.$$  

The lagged beta arises from a predictability regression of excess returns on the lagged CP factor, as discussed in Section 2.4. Note that when excess returns are unpredictable $\delta_1 = \beta^{\text{Lagged}} = 0$, and the contemporaneous CP beta is completely driven by the innovation beta. This innovation beta $\beta^{\text{Innov}}$ is what we focused on in Section in 2.2. Note also that if $CP$ follows an AR(1) process (uncorrelated with the other factors in $X$), then the weights simplify: $w_1 = \rho_{CP} = \Gamma_{(1,1)}$ and $w_2 = 1 - \rho^2_{CP}$.

Next, we derive a link between the compensation for risk in this section and in the cross-sectional exercise ($\tilde{\lambda}$). Given the expression for $\beta$ just derived and given that $E[r_{t+1}^e] = \beta \tilde{\lambda}$, it follows that

$$\tilde{\lambda} = (\Lambda_{1(1,\cdot)} \text{Cov}[X_t, CP_t] \Gamma'_{CP} + I)^{-1} \text{Var}[CP_{t+1}] \left( \Lambda_{0(1)} + \Lambda_{1(1,\cdot)} E[CP] \right).$$

If excess return are not predictable, we have $\Lambda_1 = 0$ and it immediately follows that:

$$\tilde{\lambda} = \text{Var}[CP_{t+1}] \Lambda_{0(1)}. \hspace{1cm} (5)$$

The price of CP risk from the cross-sectional exercise, which had the interpretation of a risk premium, equals the market price of risk of innovations in CP from the SDF model pre-multiplied by the variance of the CP factor.
3 Equilibrium Model

An important question that the last section leaves open is whether the SDF of section 2 can arise in an equilibrium asset pricing model. The latter SDF admits no arbitrage opportunities so that arbitrageurs cannot earn the value premium and perfectly hedge out the risk with a position in bonds. However, it does not shed light on the fundamental determinants of the yield curve and the value premium, and hence on their deeper connection. While it did a good job simultaneously describing the cross-section of stock and bond returns, the model of the previous section leaves open the question of whether there is a rational explanation for the observed covariances between $CP$, a proxy for the bond risk premium, and excess stock and bond returns. Such a model starts from preferences and technology (cash-flow growth specifications). It imposes additional discipline: it requires that the CP betas of stock returns are linked to the CP betas of bond returns and to the CP betas of dividend growth rates. We start by documenting the properties of contemporaneous CP betas for the dividend growth rates on the various stock portfolios in Section 3.1 and show that they have a distinct pattern as well. Starting in Section 3.2 we present a long-run risk (LRR) model along the lines of Bansal and Yaron (2004). Our choice of the long-run risk framework is one of analytical convenience. Because this model is well-known, we only discuss the relationships that matter for our purposes. A step-by-step model derivation is relegated to the appendix.

3.1 Cash-flow Betas in the Data

For each of the ten book-to-market portfolios, we collect data on cum- and ex-dividend returns. They allow us to construct a dividend growth series for each portfolio (see Appendix A for details). Figure 10 plots the dividend growth betas obtained from regressing dividend growth on the contemporaneous bond risk premium. In the left column, we use the $CP$ factor as a proxy for the bond risk premium, in the right panel the yield spread. While the pattern is not perfectly monotone, it does seem to be the case that growth firm’s dividend growth has a positive correlation with contemporaneous $CP$ and value firms’ dividend growth a strong negative correlation. We find a similar pattern when we use the yield spread as proxy for the bond risk premium. Dividend growth of value stocks is contemporaneously more negatively correlated with the yield spread than dividend growth of growth stocks. The patterns are similar for monthly, quarterly, and annual frequencies.

This pattern of contemporaneous dividend growth betas deepens the puzzle. The high (low) exposure of value (growth) stock returns to the CP factor does not seem to be coming from high (low) exposure of dividend growth to the CP factor. In fact, the opposite seems to be true. When

[Figure 10 about here.]
CP is high (for example, at the end of recession), current cash-flow growth on value stocks is low, while that of growth stocks is high. The model in the following sections will reconcile these facts by appealing to the behavior of future dividend growth.

3.2 Model Setup

**Technology** We modify the consumption growth specification of Bansal and Yaron (2004):

\[
\Delta c_{t+1} = \mu_c + x_t + \sigma_t \eta_{t+1}, \tag{6}
\]

\[
x_{t+1} = \rho_x x_t + \varphi_x \sigma_t e_{t+1}, \tag{7}
\]

\[
\sigma_{t+1}^2 = \hat{\sigma}^2 + \nu_1 (\sigma_t^2 - \bar{\sigma}^2) + \sigma_w \sigma_t w_{t+1}, \tag{8}
\]

\[
\Delta d_{i,t+1}^i = \mu_{d,i}^i + \phi_{d,x}^i x_t + \phi_{d,\sigma}^i (\sigma_t^2 - \bar{\sigma}^2) + \varphi_{d}^i \sigma_t u_{t+1}^i \tag{9}
\]

where \((\eta_t, e_t, w_t, u_t)\) are i.i.d. standard normal innovations. Consumption growth contains a low-frequency component \(x_t\) and is heteroscedastic, with conditional variance \(\sigma_t^2\). The two state variables \(x_t\) and \(\sigma_t^2 - \bar{\sigma}^2\) capture time-varying growth rates and time-varying economic uncertainty. Our first deviation from the canonical LRR model is that we assume that the correlation between \(e_{t+1}\) and \(w_{t+1}\) equals \(\chi \neq 0\). This creates a conditional correlation between the state variables \(x_t\) and \(\sigma_t^2 - \bar{\sigma}^2\) of \(\chi\). The second deviation is that we assume that expected dividend growth rates for portfolio \(i\) are not only a function of \(x_t\), but also of \(\sigma_t^2 - \bar{\sigma}^2\). That is, \(\phi_{d,\sigma}^i \neq 0\). Our third main deviation is that we assume that the persistence of economic uncertainty is lower than commonly assumed. We introduce two other changes which are of second-order importance. We follow Bansal and Shaliastovich (2007) and allow a correlation of \(i^i\) between temporary consumption and dividend growth shocks \(\eta_{t+1}\) and \(u_{t+1}^i\). Finally, we assume that the conditional variance of economic uncertainty moves with \(\sigma_t^2\). All other shocks at all other leads and lags are uncorrelated.

**Preferences** The long-run risk literature works off the class of preferences due to Kreps and Porteus (1978), Epstein and Zin (1989), and Duffie and Epstein (1992). These preferences impute a concern for the timing of the resolution of uncertainty. A first parameter \(\alpha\) governs risk aversion and a second parameter \(\rho\) governs the willingness to substitute consumption inter-temporally. In particular, \(\rho\) is the inverse of the inter-temporal elasticity of substitution (EIS). The appendix shows
that the log SDF $m$ has the following innovations, conditional mean, and conditional variance:

$$m_{t+1} - E_t [m_{t+1}] = -\alpha \sigma_t \eta_{t+1} - \frac{\alpha - \rho}{1 - \rho} A^c_1 \varphi_e \sigma_t e_{t+1} - \frac{\alpha - \rho}{1 - \rho} A^c_2 \sigma_w \sigma_t w_{t+1},$$

$$E_t [m_{t+1}] = m_0 - \rho x_t + \frac{\alpha - \rho}{1 - \rho} (\kappa^c_1 - \nu_1) A^c_2 \left( \sigma_t^2 - \bar{\sigma}^2 \right)$$

$$V_t [m_{t+1}] = \left[ \alpha^2 + \left( \frac{\alpha - \rho}{1 - \rho} \right)^2 \left( (A^c_1 \varphi_e)^2 + (A^c_2 \sigma_w)^2 + 2 \chi A^c_1 \varphi_e A^c_2 \sigma_w \right) \right] \sigma_t^2$$

$$m_0 = \frac{1 - \alpha}{1 - \rho} \log \beta - \frac{\alpha - \rho}{1 - \rho} \left[ \kappa^c_0 + A^c_0 (1 - \kappa^c_1) \right] - \alpha \mu_c,$$

where the log wealth-consumption ratio is given by

$$wc_t = A^c_0 + A^c_1 x_t + A^c_2 \left( \sigma_t^2 - \bar{\sigma}^2 \right),$$

and the coefficients $(A^c_0, A^c_1, A^c_2)$ solve a system of three equations in three unknowns spelled out in the appendix. The linearization constants $\kappa^c_0$ and $\kappa^c_1$ are non-linear functions of the unconditional mean wealth-consumption ratio $A^c_0$:

$$\kappa^c_1 = \frac{e^{A^c_0}}{e^{A^c_0} - 1} > 1 \quad \text{and} \quad \kappa^c_0 = -\log \left( e^{A^c_0} - 1 \right) + \frac{e^{A^c_0}}{e^{A^c_0} - 1} A^c_0. \quad (10)$$

### 3.3 Expected Excess Stock Returns

This model implies expected excess log returns (including the Jensen adjustment) given by:

$$E_t [r^{e,i}_{t+1}] = \left[ \alpha^i \varphi^i_d + \frac{\alpha - \rho}{1 - \rho} \left( A^c_1 A^i \varphi_e^2 + A^c_2 A^i_2 \sigma_w^2 + \chi (A^c_1 A^i + A^c_2 A^i_2) \varphi_e \sigma_w \right) \right] \sigma_t^2, \quad (11)$$

where the log price-dividend ratio for portfolio $i$ is given by

$$pd^i_t = A^i_0 + A^i_1 x_t + A^i_2 \left( \sigma_t^2 - \bar{\sigma}^2 \right),$$

and the coefficients $(A^i_0, A^i_1, A^i_2)$ solve a (different) system of three equations in three unknowns spelled out in the appendix. There are (different) linearization constants $\kappa^i_0$ and $\kappa^i_1$, given by equation (20), except with $A^c_0$ replaced by the unconditional mean price-dividend ratio $A^i_0$. There is one such system of equations for each portfolio $i$. When the inter-temporal elasticity of substitution exceeds unity (or $\rho < 1$), an increase in long-run growth increases the price-dividend ratio ($A^i_1 > 0$) and likewise for a decrease in economic uncertainty ($A^i_2 < 0$).

Equation (11) shows that the equity risk premium consists of a compensation for bearing temporary shock risk (first term), long-run risk (second term) and uncertainty risk (third term).
The fourth term is new and only arises when there is non-zero correlation $\chi$ between the two state variables. The market price of temporary cash-flow risk is $\alpha > 0$, the market price of long-run risk is $A_1^c \varphi_e > 0$, and the market price of uncertainty risk is $A_2^c \sigma_w < 0$. These signs make sense: when long-run risk is low (economic uncertainty is high), the marginal utility growth for the representative investor is high, and she is willing to pay a high price for a security that has high dividend growth in such states of the world. The new, fourth term in (11) lowers the equity risk premium when the state variables are positively correlated because $A_1^c A_2^c + A_2^c A_1^c < 0$. Indeed, when high uncertainty goes hand in hand with high future growth, the world is a less risky place. Finally, we note that the only source of variation in equity risk premia over time is economic uncertainty $\sigma_t^2$.

3.4 Nominal Bond Risk Premia and the CP Factor

Just like for the equity risk premium, the only source of time variation in nominal bond risk premia is economic uncertainty: $\sigma_t^2$ is a perfect predictor of future excess bond returns. Therefore, it is natural to associate the $CP$ factor with $\sigma_t^2$. In the appendix, we make this argument rigorous and provide the details. We follow Wachter (2006), Piazzesi and Schneider (2006), and Bansal and Shaliastovich (2007), and model inflation as the sum of an expected part and an unexpected part.\footnote{Unexpected inflation depends on the shocks $\eta$, $e$, and an additional shock $\xi$ which is uncorrelated with all other shocks at all leads and lags. Expected inflation depends on its own lag, on the state $x$, and its innovations also depend on $\eta$, $e$, and $\xi$.} This structure gives rise to a three-factor nominal term structure model; the factors are $x_t$, $\sigma_t^2 - \bar{\sigma}^2$, and demeaned expected inflation $\bar{\pi}_t - \mu_{\pi}$. Forward rates, like yields, are affine functions of these three state variables. Hence, any three forward rates identify the three state variables exactly at each time $t$ and a linear combination of them identifies $CP_t$: $CP_t = c_0 + c_1 f_t^8(4) + c_3 f_t^8(12) + c_5 f_t^8(20) = b_{CP}(\sigma_t^2 - \bar{\sigma}^2)$, for some constants $(c_1, c_3, c_5)$. In order to pin down the ratio of the standard deviation of $CP_t$ relative to that of $\sigma_t^2$, $b_{CP}$, we form annual expected excess bond returns on nominal bonds of horizons 2-, 3-, 4-, and 5-years in the model. They only depend on $\sigma_t^2$. We set the $CP$ factor equal to the equally-weighted average of these four returns. This is exactly how we constructed $CP$ in the data (see Section 1.1). Hence $b_{CP}$ is not a free parameter but rather a function of the structural parameters of the model.
3.5 CP Betas

The question we want to ask of the long-run risk model is whether it is able to generate the cross-sectional variation in the exposures of excess returns of book-to-market-sorted stock portfolios to the $CP$ factor. We showed in Section 3.1 that there was a clearly increasing pattern from growth to value. Since this is an equilibrium model, stock returns are endogenous, and only cash-flow growth can be specified exogenously. This leads us to impose the additional discipline that the model must match also the cross-sectional variation in the exposures of dividend growth rates of book-to-market-sorted stock portfolios to the $CP$ factor. In Section 3.1, we showed dividend growth betas that were decreasing from growth (mildly positive) to value (strongly negative). Third, the $CP$ factor is a strong predictor of bond returns in the data; we want to capture this predictability quantitatively. We now provide closed-form expressions for the contemporaneous return and dividend growth betas and for lagged-$CP$ betas for excess bond returns; the math is again relegated to the appendix.

The unconditional, contemporaneous, $CP$ beta for excess stock returns is equal to:

$$
\beta = \nu_1 \beta^\text{Lagged} + \beta^\text{Innov}
$$

$$
\beta^\text{Lagged} = \left( \frac{\phi_{i,d,\sigma} - A_1^i (\kappa_1^i - \nu_1)}{\beta_{CP}} \right)
$$

$$
\beta^\text{Innov} = \left( \frac{A_1^i \varphi_e}{\sigma_w} + A_2^i \right) \frac{1 - \nu_1^2}{\beta_{CP}}.
$$

where the expression for $\Gamma(0)$ is given in the appendix. Likewise, the contemporaneous $CP$ beta for dividend growth rates, denoted with a superscript $\Delta d$, can be shown to equal:

$$
\beta^\Delta d = \nu_1 \beta^{\Delta d,\text{Lagged}} = \nu_1 \left( \frac{\phi_{i,d,x} \varphi_e}{1 - \rho_x \nu_1} \frac{1 - \nu_1^2}{\beta_{CP}} + \frac{\phi_{i,d,\sigma}}{\beta_{CP}} \right).
$$

Finally, we find the lagged $CP$ beta for excess returns on nominal bonds of maturity $\tau$, denoted with a superscript $b$ for bonds:

$$
\beta^\text{b,Lagged}(\tau) = \frac{.5 \Gamma^g(0) - .5 \Gamma^g(\tau - 1)}{\beta_{CP}},
$$

where the expression for $\Gamma^g(\tau)$, for $\tau \geq 0$, is also given in the appendix.

An informative quantity is the difference between the contemporaneous return (not excess return) beta and the dividend growth beta. It is the difference between the $CP$ beta of news about expected (discounted) future dividend growth rates $\Delta d_{i+1}$ and the $CP$ beta of news about future
\[ \beta_{CP} - \beta_{\Delta d} = \beta \left[ C P_{t+1}, \Delta d_{t+1}^{i,H} - \kappa_1 \Delta d_{t}^{i,H} \right] - \beta \left[ C P_{t+1}, \Delta d_{t+1}^{i,H} - \kappa_1 \Delta d_{t}^{i,H} \right] \] (17)

In the data, the left hand side is positive for all portfolios and strongly increasing from growth to value. This is the result of increasing contemporaneous excess return betas and decreasing contemporaneous dividend growth betas. The Campbell-Shiller decomposition implies that this pattern must be due to either increasing CP betas of news about future cash flows (first term) or decreasing CP betas of news about future discount rates (second term). The appendix provides an expression for each in terms of the structural parameters of the model.

### 3.6 Calibration

Our goal is to understand the patterns in CP betas and returns we documented earlier. Therefore, we refrain from a wholesale re-calibration or estimation of the long-run risk model and make minimal deviations from the literature. Table 8 compares our calibration in the first column to the calibration in Bansal and Shaliastovich (2007) (Column 2) and that of the original Bansal and Yaron (2004) (Column 3). All parameters are quarterly since we solve the model at quarterly frequency and compare it to quarterly data. This avoids complications arising from time aggregation. Our parameters are identical to those in Bansal and Shaliastovich (2007), with the following exceptions. First, we introduce heteroscedasticity in \( \sigma_t^2 - \bar{\sigma}^2 \). That is, we replace \( \sigma_w \) by \( \sigma_t \) in equation (24), and multiply our value of \( \sigma_w \) by \( \bar{\sigma} \) to keep the unconditional variance of economic uncertainty the same. This change is inconsequential. Second, we estimate \( \nu_1 \), the autocorrelation coefficient of economic uncertainty \( \sigma_t^2 - \bar{\sigma}^2 \). Third, we estimate the parameter \( \chi \), which governs correlation between shocks to \( x_t \) and \( \sigma_t^2 \). Fourth, we estimate the loadings of dividend growth \( \phi_{d,\sigma}^i \) on \( \sigma_t^2 - \bar{\sigma}^2 \) and \( \phi_{d,x}^i \) on \( x_t \) for each portfolio. We use 10 “book-to-market” portfolios and 1 “market” portfolio. Prior to the estimation, we set the unconditional mean log real dividend growth rate, \( \mu_{d,t} \), equal to its observed value in the data for each of the 11 portfolios. We then estimate \( 2 + 2 \times 11 = 24 \) parameters in order to minimize the distance between model and data along the following 38 dimensions: 11 contemporaneous log excess stock return CP betas, 11 contemporaneous log real dividend growth CP betas, 11 equity risk premia, and 5 lagged bond return CP betas for maturities 1-, 2-, 5-, 7-, and 10-years. The estimation is by non-linear least squares.

[Table 8 about here.]
3.7 Benchmark Model Comparison

As a benchmark, Figure 11 shows these 38 moments for the Bansal and Shaliastovich (2007) calibration, henceforth B-S calibration. This calibration sets \( \chi = 0 \) and \( \phi_{i,d,\sigma} = 0 \). It also features a high value for \( \nu_1 \), so that economic uncertainty is very persistent, in fact more persistent than long-run growth \( (\nu_1 > \rho_x) \). To give this model its best chance at matching the value premium, we estimate \( \phi_{i,d,x} \) in order to match risk premia on each portfolio, following Bansal, Dittmar, and Lundblad (2005), Bansal, Kiku, and Yaron (2006), and Hansen, Heaton, and Li (2008). The bottom right panel shows that the model is flexible enough to exactly match not only the equity risk premium on the market portfolio but also the risk premium on each of the 10 book-to-market portfolios. The calibration accomplishes this by having value stocks carry more long-run risk: \( \phi_{i,d,x} \) is higher for value than for growth. In other words, value stocks are risky because their dividend growth is high in states of the world where long-run consumption growth \( x_t \) is high. These are states of low marginal utility growth for the representative investor. Panel A of Figure 11 shows that \( \phi_{i,d,x} \) ranges from 3.3 for BM1 to 5.3 for BM10. It is 3.5 for the market portfolio. A higher \( \phi_{i,d,x} \) translates in a higher \( A_{i,1} \), which increases the equity risk premium through the second term in equation (11).

However, inspection of the two top panels of Figure 11 shows that this calibration fails to generate the observed relationship between the \( CP \) factor and excess stock returns and dividend growth rates. First, equation (15) shows that when both \( \phi_{i,d,\sigma} = 0 \) and \( \chi = 0 \), the contemporaneous dividend growth betas are zero for all portfolios. Because expected dividend growth does not depend on economic uncertainty \( \sigma_t^2 \), it is uncorrelated with \( CP \) (which perfectly correlates with economic uncertainty). Second, the excess stock return betas do not show the increasing pattern we found in the data, and they are at least a factor of five too big. Equation (12) helps us understand why. The innovation betas are all negative because \( \chi = 0 \) and \( A_{i,2} < 0 \). They become increasingly negative from growth to value: from -8 to -16. The lagged excess return betas in (13) are the sum of \( \phi_{i,d,\sigma} - A_{i,2} (\kappa_1^c - \nu_1) \), which increases from 360 to 630 from growth to value, and \( \frac{\sigma - \rho}{1 - \rho} A_{i,c} (\kappa_1^c - \nu_1) + 0.5 \Gamma(0) = -993 + 1,046 = 53 \). Given \( b_{CP} = 35 \), when \( \phi_{i,d,\sigma} = 0 \), these lagged excess stock return betas range from 11 to 19. Combining the two results in contemporaneous betas that are too high without a pronounced pattern from growth to value. Finally, he first beta on the right hand side of equation (17) is zero when \( \chi = 0 \) and \( \phi_{i,d,\sigma} = 0 \). The second term is negative: high bond risk premia (or positive innovations in \( CP \)) coincide with negative innovations in expected future stock returns, which seems counter-intuitive. In addition, there is no clear pattern in this beta difference as opposed to the strongly increasing pattern in the data.
3.8 Results for Our Calibration

Our estimation sets $\chi = .2048 > 0$ and $\phi_{d,\sigma}^i < 0$ in order to match the 38 moments. It also substantially lowers the value for $\nu_1$ from .9880 to .9296 ($\nu_1 < \rho_x$). The lower persistence we estimate in economic uncertainty facilitates the interpretation of $\sigma_t^2$ (or $CP_t$) as a business cycle variable. Figure 12 shows that our calibration does a good job matching the patterns and magnitudes of the CP exposures of excess stock returns, dividend growth rates, while maintaining the good fit for the value premium. What drives these results? First, a sufficiently negative $\phi_{d,\sigma}^i$ directly generates a negative dividend growth beta. Panel B of Table 4 shows that the estimation chooses negative loadings $\phi_{d,\sigma}^i$ for all portfolios. They range from -537 for growth (BM1) and decrease monotonically to -883 for value (BM10). According to this channel, stocks, and in particular value stocks, have lower expected dividend growth when economic uncertainty, and therefore $CP$, is high. Second, this makes them risky because they have low cash flow growth exactly when marginal utility growth is high. Hence the value premium. This $\phi_{d,\sigma}^i$ effect reinforces the $\phi_{d,x}^i$ effect. Panel B shows that value stocks continue to have higher loadings on long run growth $x_t$ than growth stocks, just as in Panel A. The dividend growth properties of the market portfolio resemble those of the third growth decile portfolio, consistent with the finding in Jurek and Viceira (2006) that growth stocks make up about 70% of the U.S. market capitalization.

Because $\chi$ is estimated to be .20, $x_t$ and $\sigma_t^2$ have a conditional correlation of 20%. Positive correlation between the two state variables implies that times with higher growth are also times with more economic uncertainty. Such an economy is less risky relative to the one with $\chi = 0$ because the shocks to uncertainty (which causes prices to fall) are hedged by a higher long-term growth rate. The interest rate increases (slightly) because the demand for bonds is lower because of reduced precautionary savings motives. Because the economy is less risky, the price dividend ratios on all assets are higher and the risk premia lower. This reduction in risk is why all $\phi_{d,x}^i$ values are somewhat higher in Panel B ($\chi = 0.2$) than in Panel A ($\chi = 0$). Third, the positive $\chi$ helps to increase the excess stock return innovation betas, while the negative $\phi_{d,\sigma}^i$ helps to lower the lagged excess stock return betas. Both help to match the observed magnitude of the contemporaneous excess stock return betas. A positive $\chi$ suggests that economic uncertainty is highest at the end of a recession, when growth is about to resume. Such a positive correlation arises endogenously in models of information production (Van Nieuwerburgh and Veldkamp 2006). It also increases the correlation between expected returns and expected dividend growth. (Binsbergen and Koijen 2007) estimate this correlation for the aggregate stock market and conclude it is positive. In both our model and the B-S calibration, the lagged excess bond return betas are somewhat too high. This is not surprising, in the model bond returns and $CP$ are both driven by $\sigma_t^2$, with no offsetting effect from cash-flow growth as in the case of stocks. Finally, our calibration predicts a strongly increasing pattern in the beta difference of equation (17), just as in the data. The first beta on
the right hand side is positive and increasing from growth to value. A high bond risk premium (at the end of a recession or the beginning of a boom) implies good news about future dividend growth, and more so for value stocks than for growth stocks. The second term is also positive and increasing. A high bond risk premium (at the end of a recession) coincides with a high equity risk premium, and more so for value than for growth stocks. The cash-flow effect dominates the discount rate effect.

Matching these betas does not come at the expense of matching the moments of consumption, interest rates, or inflation, for the most part because we kept these parameters at their standard values. Table 10 plots unconditional means, standard deviations (both expressed in percent per year), and quarterly autocorrelation in the benchmark B-S calibration (first 3 columns) and in our calibration (last 3 columns). The moments in our model are both close to the ones in the B-S calibration and close to the data.

**Discussion** The higher loading of value portfolios’ returns on the bond risk factor (CP) are puzzling at first thought. Long-maturity bonds are more sensitive to the bond risk factor than short maturity bonds (one-year excess returns on 5-year bonds have a higher sensitivity to CP and yield spread than one-year excess returns on 2-year bonds). In the absence of priced cash-flow risk, the only difference between various stocks is the timing (duration) of their cash flows. Lettau and Wachter (2007b) think of growth stocks as long duration equity and value stocks as short duration equity. For the same reason that long-maturity bonds are more sensitive to the bond risk premium than short maturity bonds, a *growth risk premium* would arise. In order to explain the value premium, then, future cash flows on value stocks must be riskier than those on growth stocks to offset this discount rate effect. This is exactly what we find above, based on our estimation of the CP return and dividend growth exposures. The deeper question is why future dividend growth on value and growth stocks have this covariance pattern with the bond risk premium. One potential explanation for the higher cash-flow risk of value firms is that they have cash-flow growth that is more cyclical than that of growth firms: expected future dividend growth on value stocks in relatively low at the start of a recession, when CP is low, and relatively high at the end of a recession, when CP is high. This itself could be due to differences in the prevalence of assets-in-place versus growth options or in the maturity structure of debt. This is beyond the scope of the current paper.
4 Conclusion

We document that differential exposure to the Cochrane-Piazzesi (CP) factor, a strong predictor of future excess bond returns and hence a proxy for the bond risk premium, can help explain why value stocks have higher average returns than value stocks. This connection between a bond market and the cross-section of stocks leads us to develop a unified pricing model for stocks and bonds. We propose a no-arbitrage stochastic discount factor model that combines the canonical three-factor affine term structure model (level, slope, and curvature) with the CP factor, and an aggregate stock market return factor familiar from the CAPM. Only three factors need to have non-zero prices of risk to account for average returns on the book-to-market decile portfolios, the market portfolio, and maturity-sorted government bond portfolios. The price of market risk captures the common level of equity risk premia, the price of level risk captures the cross-sectional spread between long-term and short-term bond returns, and the price of CP risk captures the cross-sectional spread between value and growth stock returns. This model also captures the dynamics of bond yields and expected excess stock returns well when the market prices of risk change with the CP factor. We explore what sources of underlying economic risk the CP factor captures in the context of the long-run risk model. We show that this equilibrium asset pricing model is able to replicate the differential exposure of excess stock returns to the CP factor of value and growth stocks. The CP factor is high at the end of a recession, periods that are characterized by high economic uncertainty but also good long-term growth prospects. A value-minus-growth strategy has high realized returns in periods when CP is high because value stocks receive good news about future cash flow growth in such periods.
References


A Constructing Quarterly Dividend Growth

To avoid seasonality in monthly (quarterly) data, we form the dividend-price ratio as the ratio of the equally-weighted average of the current dividend and the dividends over the past 11 months (3 quarters) and the current price. We define dividend growth correspondingly so that the adjusted dividend growth rate and change in the dividend price ratio gives back the original cum-dividend return. In symbols, cum-dividend returns equal $R_{t+1}^\text{ex} = \frac{P_{t+1}^\text{ex}}{P_t}$, where $DG_{t+1} = D_{t+1}/D_t$ and $PD_t = R_t/D_t$. Define the ex-dividend return as $R_{t+1}^\text{ex}$ and construct the dividend yield as the difference between the cum- and ex-dividend return: $DY_{t+1} = R_{t+1} - R_{t+1}^\text{ex}$. This leads to the quarterly dividend $D_{t+1} = DY_{t+1} + P_t^\text{ex}$. The ex-dividend price index is updated recursively: $P_t^\text{ex} = P_t^\text{ex}(1 + R_t^\text{ex})$. For quarterly data, the adjusted variables, denoted with a tilde, are constructed as follows. The adjusted capital gain $\tilde{R}_{t+1}^\text{ex} = \frac{\tilde{P}_{t+1}^\text{ex}}{\tilde{P}_t^\text{ex}} - (D_{t-2}/4 + D_{t-1}/4 + D_t/4 - 3D_{t+1}/4)/P_t^\text{ex}$. Of course, $\tilde{P}D_t = (\tilde{DP}_t)^{-1}$. One can verify that this construction leaves gross returns unchanged: $R_{t+1} = \frac{\tilde{DP}_{t+1}}{\tilde{P}_t}$. The construction of monthly data is similar.

B Long-Run Risk Model in Detail

B.1 Setup

Preferences The long-run risk literature works off the class of preferences due to Kreps and Porteus (1978), Epstein and Zin (1989), and Duffie and Epstein (1992). These preferences impute a concern for the timing of the resolution of uncertainty. A first parameter $\alpha$ governs risk aversion and a second parameter $\rho$ governs the willingness to substitute consumption inter-temporally. In particular, $\rho$ is the inverse of the inter-temporal elasticity of substitution (EIS). Bansal and Yaron (2004) show that the stochastic discount factor can be written as a function of consumption growth and the return to the wealth portfolio:

$$M_{t+1} = \beta^{\frac{1 + \alpha}{1 + \rho}} \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \left( \frac{R_{t+1}^c}{1} \right)^{\frac{\alpha}{1 + \rho}}.$$  \hspace{1cm} (18)

The return on a claim to aggregate consumption, the total wealth return, can be written as

$$R_{t+1}^c = \frac{W_{t+1}}{W_t - C_t} = \frac{C_{t+1}}{C_t} \frac{WC_{t+1}}{WC_t - 1}.$$  \hspace{1cm}

We start by using the Campbell (1991) approximation of the log total wealth return $r_t^c = \log(R_t^c)$ around the long-run average log wealth-consumption ratio $A_0^c = E[w_t - c_t]$:

$$r_{t+1}^c = \kappa_0^c + \Delta c_{t+1} + wc_{t+1} - \kappa_t wc_t,$$  \hspace{1cm} (19)

where we define the log wealth-consumption ratio $wc$ as

$$wc_t = \log \left( \frac{W_t}{C_t} \right) = w_t - c_t.$$  \hspace{1cm}

Throughout, variables with a subscript zero denote unconditional averages.
The linearization constants \( \kappa_0^c \) and \( \kappa_1^c \) are non-linear functions of the unconditional mean wealth-consumption ratio \( A_0^e \):

\[
\kappa_1^c = \frac{e^{A_0^c}}{e^{A_0^c} - 1} > 1 \quad \text{and} \quad \kappa_0^c = -\log \left( \frac{e^{A_0^c}}{e^{A_0^c} - 1} \right) + \frac{e^{A_0^c}}{e^{A_0^c} - 1} A_0^c.
\]

Using the definition of the total wealth return, one can then show that the log SDF becomes:

\[
m_{t+1} = \frac{1 - \alpha}{1 - \rho} \left[ \log(\beta + \kappa_0^c) - \kappa_0^c - \alpha \Delta c_{t+1} - \frac{\alpha - \beta}{1 - \rho} (w_{c,t+1} - \kappa_1^c w_c) \right].
\]

### Technology

We mostly adopt the consumption growth specification of Bansal and Yaron (2004):

\[
\begin{align*}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_c \eta_{t+1}, \quad (22) \\
x_{t+1} &= \rho x_t + \varphi_c \sigma_t c_{t+1}, \quad (23) \\
\sigma_{t+1}^2 &= \bar{\sigma}^2 + \nu_t (\sigma_t^2 - \bar{\sigma}^2) + \sigma_w \sigma_t w_{t+1}, \quad (24)
\end{align*}
\]

where \( (\eta_t, c_t, w_t) \) are i.i.d. standard normal innovations. Consumption growth contains a low-frequency component \( x_t \) and is heteroscedastic, with conditional variance \( \sigma_t^2 \). The two state variables \( x_t \) and \( \sigma_t^2 - \bar{\sigma}^2 \) capture time-varying growth rates and time-varying economic uncertainty.

The two changes we make relative to Bansal and Yaron (2004) are that (1) the innovation in equation (22) is not \( \sigma_w w_{t+1} \) but \( \sigma_w \sigma_t w_{t+1} \), and (2) we assume that \( x_t \) and to \( \sigma_t^2 - \bar{\sigma}^2 \) have a non-zero conditional correlation \( \chi \). We engineer this through a non-zero unconditional correlation \( \chi \) between the contemporaneous shocks \( c_{t+1} \) and \( w_{t+1} \):

\[
\text{Corr}[\epsilon_{t+1}, w_{t+1}] = \text{Cov}[\epsilon_{t+1}, w_{t+1}] = E[\epsilon_{t+1} w_{t+1}] = \chi.
\]

All other shocks have zero correlation with each other at all leads and lags. To find the correlation between \( x_t \) and to \( \sigma_t^2 - \bar{\sigma}^2 \), start by working out

\[
E_t[x_{t+1}(\sigma_{t+1}^2 - \bar{\sigma}^2)] = E_t[(\rho_x x_t + \varphi \sigma_t c_{t+1}) (\nu_t (\sigma_t^2 - \bar{\sigma}^2) + \sigma_w \sigma_t w_{t+1})],
\]

\[
= \rho_x \nu_t x_t (\sigma_t^2 - \bar{\sigma}^2) + \rho \rho_x x_t \sigma_t \sigma_w E_t[w_{t+1}],
\]

\[
+ \varphi \sigma_t \nu_t (\sigma_t^2 - \bar{\sigma}^2) E_t[c_{t+1}] + \varphi \sigma_t^2 \sigma_w E_t[w_{t+1} w_{t+1}],
\]

\[
= \rho_x \nu_t x_t (\sigma_t^2 - \bar{\sigma}^2) + \varphi \sigma_w \chi \sigma_t^2.
\]

The conditional covariance between \( x_{t+1} \) and \( \sigma_{t+1}^2 - \bar{\sigma}^2 \) equals:

\[
\text{Cov}_t[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] = E_t[x_{t+1}(\sigma_{t+1}^2 - \bar{\sigma}^2)] - E_t[x_{t+1}]E_t[(\sigma_{t+1}^2 - \bar{\sigma}^2)],
\]

\[
= \rho_x \nu_t x_t (\sigma_t^2 - \bar{\sigma}^2) + \varphi \sigma_w \chi \sigma_t^2 - \rho \nu_t x_t (\sigma_t^2 - \bar{\sigma}^2),
\]

\[
= \varphi \sigma_w \chi \sigma_t^2.
\]

It follows that the conditional correlation between \( x_{t+1} \) and \( \sigma_{t+1}^2 - \bar{\sigma}^2 \) is also \( \chi \):

\[
\text{Corr}_t[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] = \frac{\text{Cov}_t[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2]}{\text{Std}_t[x_{t+1}] \text{Std}_t[\sigma_{t+1}^2 - \bar{\sigma}^2]} = \frac{\chi \varphi \sigma_w \chi \sigma_t^2}{\varphi \sigma_w \sigma_t^2} = \chi.
\]

36
The unconditional variance of $x_{t+1}$ and $\sigma_t^2 - \bar{\sigma}^2$ is:

$$V[x_{t+1}] = E[x_{t+1}^2] = \frac{\varphi_2^2 \bar{\sigma}^2}{1 - \rho_x^2},$$

$$V[\sigma_{t+1}^2 - \bar{\sigma}^2] = E[(\sigma_{t+1}^2 - \bar{\sigma}^2)^2] = \frac{\sigma_w^2 \bar{\sigma}^2}{1 - \nu_t^2}.$$ 

The unconditional covariance between $x_{t+1}$ and $\sigma_{t+1}^2 - \bar{\sigma}^2$ equals:

$$\text{Cov}[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] = \frac{\chi \varphi_e \sigma_w \bar{\sigma}^2}{1 - \rho_x \nu_1}.$$ 

The unconditional correlation between $x_{t+1}$ and $\sigma_{t+1}^2 - \bar{\sigma}^2$ equals:

$$\text{Corr}[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] = \frac{\text{Cov}[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2]}{\text{Std}[x_{t+1}] \text{Std}[\sigma_{t+1}^2 - \bar{\sigma}^2]},$$

$$= \frac{\chi \varphi_e \sigma_w \bar{\sigma}^2}{\sqrt{1 - \rho_x^2} \sqrt{1 - \nu_t^2}} = \chi \frac{\sqrt{1 - \rho_x^2} \sqrt{1 - \nu_t^2}}{1 - \rho_x \nu_1}. $$

The unconditional correlation between these two state variables is zero at all other leads and lags.

**Dividend Growth** We specify dividend growth processes for various assets or portfolios $i$, making two important modifications to the specification in Bansal and Yaron (2004):

$$\Delta d_{t+1}^i = \mu_d^i + \phi_{d,x}^i x_t + \phi_{d,\sigma}^i (\sigma_t^2 - \bar{\sigma}^2) + \varphi_d^i \sigma_t u_{t+1}^i $$

(25)

In Bansal and Yaron (2004), the shock $u_t$ is assumed orthogonal to $(\eta, e, w)$ and $\phi_d^i = 0$. Then, correlation between consumption and dividend growth comes through the two state variables $x_t$ and $\sigma_t$. Instead, we allow for a correlation $\iota$ between $\eta_{t+1}$ and $u_{t+1}^i$. All other correlations are kept at zero. This opens up an additional channel of correlation and is a modification also made by (Bansal and Shaliastovich 2007). It turns out to be immaterial for our purposes. Much more important is the non-zero loading of dividend growth on economic uncertainty $\phi_d^i = 0$. Its role will become clear below.

Defining returns ex-dividend and using the Campbell (1991) linearization, the log return on a claim to the dividend of portfolio $i$ can be written as:

$$r_{t+1}^i = \Delta d_{t+1}^i + pd_{t+1}^i + \kappa_0^i - \kappa^i pd_{t+1}^i,$$

with coefficients

$$\kappa_1^i = \frac{e^{A_0^i}}{e^{A_0^i} - 1} > 1, \text{ and } \kappa_0^i = -\log \left( e^{A_0^i} - 1 \right) + \frac{e^{A_0^i}}{e^{A_0^i} - 1} A_0^i,$$

which depend on the long-run log price-dividend ratio $A_0^i$.

**Inflation** We follow Wachter (2006), Piazzesi and Schneider (2006), and Bansal and Shaliastovich (2007), and model inflation as

$$\pi_{t+1} = \tilde{\pi}_t + \varphi_{\pi, \eta} \sigma_t \eta_{t+1} + \varphi_{\pi, e} \varphi_e \sigma_t e_{t+1} + \sigma_{\pi} \xi_{t+1},$$

(26)

where expected inflation $\tilde{\pi}_t = E_t[\pi_{t+1}]$ is given by

$$\tilde{\pi}_{t+1} = \mu_{\pi} + \rho_{\pi}(\tilde{\pi}_t - \mu_{\pi}) + \phi_{\pi} x_t + \zeta_{\pi, \eta} \sigma_t \eta_{t+1} + \zeta_{\pi, e} \varphi_e \sigma_t e_{t+1} + \sigma_{\pi} \xi_{t+1},$$

(27)
The innovation $\xi_t$ is orthogonal to all other innovations ($\eta_t, e_t, w_t, u_t^i$). Expected inflation mean-reverts, it carries long-run risk with loading $\phi_\pi$, and its innovations are correlated with unexpected inflation, and with the temporary and persistent component of consumption growth.

The nominal SDF is given by:

$$m^s_{t+1} = m_{t+1} - \pi_{t+1}.$$ 

Expected inflation (demeaned), $\bar{\pi}_t - \mu_\pi$, is a third state variable that drives the nominal yield curve. We are interested in understanding its variance and covariance with the other two state variables $x_{t+1}$ and $\sigma_t^2 - \bar{\sigma}^2$. First, we calculate

$$Cov[x_{t+1}, \bar{\pi}_{t+1} - \mu_\pi] = \rho_x \rho_\pi Cov[x_t, \bar{\pi}_t - \mu_\pi] + \rho_x \phi_\pi \bar{\pi}_t + \zeta_{\pi,e} \varphi_e \varphi_\pi \bar{\sigma}^2,$$

$$= \frac{\rho_x \phi_\pi + \zeta_{\pi,e} \varphi_e \varphi_\pi}{1 - \rho_x \rho_\pi} \bar{\sigma}^2.$$

The covariance with economic uncertainty is

$$Cov[\sigma_t^2 - \bar{\sigma}^2, \bar{\pi}_{t+1} - \mu_\pi] = \nu_1 \rho_x Cov[\sigma_t^2 - \bar{\sigma}^2, \bar{\pi}_t - \mu_\pi] + \nu_1 \phi_\pi Cov[\sigma_t^2 - \bar{\sigma}^2] + \chi \zeta_{\pi,e} \varphi_e \sigma_w \bar{\sigma}^2,$$

$$= \frac{\nu_1 \phi_\pi + \zeta_{\pi,e} \varphi_e \sigma_w}{1 - \nu_1 \rho_\pi} \bar{\sigma}^2.$$

We can now calculate the unconditional variance of expected inflation

$$E[(\bar{\pi}_{t+1} - \mu_\pi)^2] = \rho_\pi^2 E[(\bar{\pi}_t - \mu_\pi)^2] + \phi_\pi^2 E[x_{t+1}^2] + \zeta_{\pi,\eta}^2 \bar{\sigma}^2 + \zeta_{\pi,e}^2 \varphi_e \bar{\sigma}^2 + \sigma_e^2 + 2 \phi_\pi \rho_\pi \bar{\pi}_t - \mu_\pi,$$

$$= \frac{\zeta_{\pi,\eta}^2 \bar{\sigma}^2 + \frac{\phi_\pi^2}{1 - \rho_\pi^2} + \frac{\zeta_{\pi,e}^2}{1 - \rho_\pi^2} + 2 \phi_\pi \rho_\pi \bar{\pi}_t}{1 - \rho_\pi^2} \varphi_e \bar{\sigma}^2 + \sigma_e^2.$$

### B.2 Equity pricing

**Euler Equation** The starting point of the analysis is the Euler equation $E_t[M_{t+1} R_{t+1}^i] = 1$, where $R_{t+1}^i$ denotes a gross return between dates $t$ and $t+1$ on some asset $i$ and $M_{t+1}$ is the SDF. In logs:

$$E_t[m_{t+1} + E_t[r_{t+1}^i] + \frac{1}{2} Var_t[m_{t+1}] + \frac{1}{2} Var_t[r_{t+1}^i] + Cov_t[m_{t+1}, r_{t+1}^i] = 0. \quad (28)$$

The same equation holds for the real risk-free rate $y_t(1)$, so that

$$y_t(1) = -E_t[m_{t+1}] - \frac{1}{2} Var_t[m_{t+1}]. \quad (29)$$

The expected excess return becomes:

$$E_t[r_{t+1}^{e,i}] = E_t[r_{t+1}^{i} - y_t(1)] + \frac{1}{2} Var_t[r_{t+1}^{i}] = -Cov_t[m_{t+1}, r_{t+1}^{i}] = -Cov_t[m_{t+1}, r_{t+1}^{e,i}], \quad (30)$$

where $r_{t+1}^{e,i}$ denotes the excess return on asset $i$ corrected for the Jensen term.

**The Consumption Claim** In what follows we focus on the return on a claim to aggregate consumption, denoted $r^c$, where

$$r_{t+1}^c = \kappa_0^c + \Delta c_{t+1} + wc_{t+1} - \kappa_1^c wc_t,$$
and derive the five terms in equation (28) for this asset.

Taking logs on both sides of the non-linear SDF expression in equation (13) delivers an expression of the log SDF as a function of log consumption changes and the log total wealth return

\[ m_{t+1} = \frac{1 - \alpha}{1 - \rho} \log \beta - \frac{1 - \alpha}{1 - \rho} \rho \Delta c_{t+1} + \left( \frac{1 - \alpha}{1 - \rho} - 1 \right) r^c_{t+1}. \] (31)

We conjecture that the log wealth-consumption ratio is linear in the two states \( x_t \) and \( \sigma_t^2 - \bar{\sigma}^2 \),

\[ wc_t = A_0^c + A_1^c x_t + A_2^c \left( \sigma_t^2 - \bar{\sigma}^2 \right). \]

As BY, we assume joint conditional normality of consumption growth, \( x_t \), and the variance of consumption growth. We verify this conjecture from the Euler equation (28).

Using the conjecture for the wealth-consumption ratio, we compute innovations in the total wealth return, and its conditional mean and variance:

\[
\begin{align*}
E_t [r^c_{t+1}] &= \sigma_t \eta_{t+1} + A_1^c \varphi_c \sigma_t \epsilon_{t+1} + A_2^c \sigma_w \sigma_t \omega_{t+1} \\
E_t [r^c_{t+1}] &= r_0 + (1 - (\kappa_1^c - \rho_x) A_1^c) x_t - A_2^c (\kappa_1^c - \nu_1) (\sigma_t^2 - \bar{\sigma}^2) \\
V_t [r^c_{t+1}] &= [1 + (A_1^c \varphi_c)^2 + (A_2^c)^2 \sigma_w^2 + 2 A_1^c \varphi_c A_2^c \sigma_w \chi] \sigma_t^2 \\
r_0 &= \kappa_0^c + A_0^c (1 - \kappa_1^c) + \mu_c
\end{align*}
\]

Substituting in the expression for the log total wealth return \( r^c \) into the log SDF, we compute innovations, and the conditional mean and variance of the log SDF:

\[
\begin{align*}
m_{t+1} - E_t [m_{t+1}] &= -\alpha \sigma_t \eta_{t+1} - \frac{\alpha - \rho}{1 - \rho} A_1^c \varphi_c \sigma_t \epsilon_{t+1} - \frac{\alpha - \rho}{1 - \rho} A_2^c \sigma_w \sigma_t \omega_{t+1} \\
E_t [m_{t+1}] &= m_0 + \left( -\alpha + \frac{\alpha - \rho}{1 - \rho} A_1^c (\kappa_1^c - \rho_x) \right) x_t + \frac{\alpha - \rho}{1 - \rho} (\kappa_1^c - \nu_1) A_2^c (\sigma_t^2 - \bar{\sigma}^2) \\
V_t [m_{t+1}] &= \left[ \alpha^2 + \left( \frac{\alpha - \rho}{1 - \rho} \right)^2 \left( (A_1^c \varphi_c)^2 + (A_2^c \sigma_w)^2 + 2 \chi A_1^c \varphi_c A_2^c \sigma_w \chi \right) \right] \sigma_t^2 \\
m_0 &= 1 - \frac{\alpha}{1 - \rho} \log \beta - \frac{\alpha - \rho}{1 - \rho} \left( \kappa_0^c + A_0^c (1 - \kappa_1^c) \right) - \alpha \mu_c
\end{align*}
\] (32)

The conditional covariance between the log consumption return and the log SDF is given by the conditional expectation of the product of their innovations

\[
\begin{align*}
\text{Cov}_t [r^c_{t+1}, m_{t+1}] &= E_t [r^c_{t+1} - E_t [r^c_{t+1}], m_{t+1} - E_t [m_{t+1}]] \\
&= \left[ -\alpha - \frac{\alpha - \rho}{1 - \rho} \left( (A_1^c \varphi_c)^2 + (A_2^c \sigma_w)^2 + 2 \chi A_1^c \varphi_c A_2^c \sigma_w \chi \right) \right] \sigma_t^2
\end{align*}
\]

Using the method of undetermined coefficients and the five components of equation (28), we can solve for the
The unconditional variance of these quantities is:

\[ \sigma^2 = \frac{1 - \rho}{\kappa^2 - \rho_x}, \]

\[ A^e_1 = \frac{1 - \rho}{\kappa_1^e - \rho_x}, \quad A^e_2 = \frac{1 - \rho}{\kappa_1^e - \rho_1} \left[ 1 + \frac{(A^e_1 \varphi_c)^2 + (A^e_2 \sigma_w)^2 + 2\chi A^e_1 \varphi_c A^e_2 \sigma_w}{(1 - \rho)^2} \right], \]

\[ 0 = \frac{1 - \rho}{1 - \rho} \left[ \log \beta + \kappa_0^e + (1 - \kappa_1^e)A_0^e \right] + \alpha \mu_c + \frac{1}{2} \left[ 1 + \frac{(A^e_1 \varphi_c)^2 + (A^e_2 \sigma_w)^2 + 2\chi A^e_1 \varphi_c A^e_2 \sigma_w}{(1 - \rho)^2} \right] \sigma_t^2 \quad \text{(34)} \]

The first equation is the same as in the case of no correlation between the state variables. The second and third equations are implicit functions of \( A_0^e \) and \( A_1^e \). Because \( \kappa_0^e \) and \( \kappa_1^e \) are non-linear functions of \( A_0^e \), this system of three equations needs to be solved simultaneously and numerically. Our computations indicate that the system has a unique solution. This verifies the conjecture that the log wealth-consumption ratio is linear in the two state variables.

**The Consumption Risk Premium** According to (30), the risk premium (expected excess return corrected for a Jensen term) on the consumption claim is given by

\[ E_t[r^c_{t+1}] = -\text{Cov}_t[r^c_{t+1}, m_{t+1}] \]

\[ = [\alpha + \frac{\alpha - \rho}{1 - \rho} \left\{ (A^c_1 \varphi_c)^2 + (A^c_2 \sigma_w)^2 + 2\chi A^c_1 \varphi_c A^c_2 \sigma_w \right\}] \sigma_t^2 \]

\[ = (\lambda_{m,n} + \lambda_{m,e} A^c_1 \varphi_e + \lambda_{m,w} A^c_2 \sigma_w + 2\chi \lambda_m \lambda \lambda \sigma_t^2), \]

with the market price of risk vector \( \Lambda = [\lambda_{m,n}, \lambda_{m,e}, \lambda_{m,w}] \) given by

\[ \lambda_{m,n} = \alpha, \]

\[ \lambda_{m,e} = \frac{\alpha - \rho}{1 - \rho} A^c_1 \varphi_e = \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \varphi, \]

\[ \lambda_{m,w} = \frac{\alpha - \rho}{1 - \rho} A^c_2 \sigma_w. \]

Note that the market price of temporary consumption growth shocks \( \eta_{t+1} \) is positive and equal to the risk aversion coefficient \( \alpha \), the market price of long-run consumption growth shocks \( \epsilon_{t+1} \) is positive, and the market price of economic uncertainty (innovations \( \omega_{t+1} \) in \( \sigma^2 - \bar{\sigma}^2 \)) is negative because \( A^e_2 \) will turn out to be negative.

**Correlations expected consumption growth and expected total wealth return** Expected total wealth returns and expected consumption growth rates are equal to:

\[ E_t[r^c_{t+1}] = r_0^c + \rho x_t - A^e_2 (\kappa_1^c - \nu_1) (\sigma_t^2 - \bar{\sigma}^2), \]

\[ E_t[\Delta c_{t+1}] = \mu_c + x_t. \]

The unconditional variance of these quantities is:

\[ V[E_t[r^c_{t+1}]] = \rho^2 V[x_t] + [A^e_2 (\kappa_1^c - \nu_1)]^2 V[\sigma_t^2 - \bar{\sigma}^2] - 2\rho A^e_2 (\kappa_1^c - \nu_1) \text{Cov}[x_t, \sigma_t^2 - \bar{\sigma}^2], \]

\[ V[E_t[\Delta c_{t+1}]] = V[x_t]. \]
Finally, the conditional covariance between the log SDF and the log dividend claim return is

\[
\text{Cov} \left( E_t \left[ r_{t+1}^i \right], E_t \left[ \Delta c_{t+1} \right] \right) = \rho V[x] - A_2^i (\kappa_1^i - \nu_1) \text{Cov} \left[ x_t, \sigma_t^2 - \bar{\sigma}^2 \right]
\]

This then leads to the unconditional correlation, which is straightforward to compute.

**The Dividend Claims**  We conjecture, as we did for the wealth-consumption ratio, that the log price dividend ratio on a stock \( i \), where the notation \( i \) is suppressed, is linear in the two state variables:

\[ pd_t^i = A_0^i + A_1^i x_t + A_2^i (\sigma_t^2 - \bar{\sigma}^2). \]

As we did for the return on the consumption claim, we compute innovations in the dividend claim return, and its conditional mean and variance:

\[
\begin{align*}
 r_{t+1}^i - E_t \left[ r_{t+1}^i \right] &= \varphi_d^i \sigma_t v_{t+1} + A_1^i \varphi_c \sigma_t v_{t+1} + A_2^i \sigma_t \sigma_t v_{t+1} \\
 E_t \left[ r_{t+1}^i \right] &= r_0^i + [\phi_{d,x}^i - A_1^i (\kappa_1^i - \rho_x)] x_t + [\phi_{d,\sigma}^i - A_2^i (\kappa_1^i - \nu_1)] (\sigma_t^2 - \bar{\sigma}^2) \\
 V_t \left[ r_{t+1}^i \right] &= \left[ (\varphi_d^i)^2 + (A_1^i \varphi_c)^2 + (A_2^i \sigma_t)^2 + 2 \chi A_1^i \varphi_c A_2^i \sigma_t \right] \sigma_t^2, \\
 r_0^i &= \kappa_0^i + A_0^i (1 - \kappa_1^i) + \mu_0^i
\end{align*}
\]

Finally, the conditional covariance between the log SDF and the log dividend claim return is

\[
\text{Cov}_t \left[ m_{t+1}, r_{t+1}^i \right] = - \left[ \varphi_d^i + \frac{\alpha - \rho}{1 - \rho} \left\{ A_1^i A_1^i \varphi_c^2 + \chi A_1^i A_2^i \varphi_c \sigma_t + \chi A_2^i A_1^i \varphi_c \sigma_t + A_2^i A_2^i \sigma_t^2 \right\} \right] \sigma_t^2.
\]

From the Euler equation for this return \( E_t \left[ m_{t+1} + E_t \left[ r_{t+1}^i \right] + \frac{1}{2} V_t \left[ m_{t+1} + V_t \left[ r_{t+1}^i \right] + \text{Cov}_t \left[ m_{t+1}, r_{t+1}^i \right] \right] = 0 \) and the method of undetermined coefficients, we can use the same procedure as above, and solve for the constants \( A_0^i \), \( A_1^i \), and \( A_2^i \):

\[
\begin{align*}
 A_1^i &= \frac{\phi_{d,x}^i - \rho}{\kappa_1^i - \rho_x}, \\
 A_2^i &= \left[ \frac{\alpha - \rho}{1 - \rho} A_2^i (\kappa_1^i - \nu_1) + \phi_{d,\sigma}^i + 0.5 H_i \right], \\
 0 &= m_0 + \kappa_0^i + A_0^i (1 - \kappa_1^i) + \mu_0^i + 0.5 H_i \bar{\sigma}^2
\end{align*}
\]

where

\[
H_i = \left\{ \begin{array}{l}
\alpha^2 + (\varphi_d^i)^2 - 2 \varphi_d^i \varphi_c^i + \left\{ (A_1^i \varphi_c)^2 + (A_2^i \sigma_t)^2 + 2 \chi A_1^i \varphi_c A_2^i \sigma_t \right\} \\
+ \left( \frac{\alpha - \rho}{1 - \rho} \right)^2 \left\{ (A_1^i \varphi_c)^2 + (A_2^i \sigma_t)^2 + 2 \chi A_1^i \varphi_c A_2^i \sigma_t \right\} \\
- 2 \frac{\alpha - \rho}{1 - \rho} \left\{ A_1^i A_1^i \varphi_c^2 + \chi A_1^i A_2^i \varphi_c \sigma_t + \chi A_2^i A_1^i \varphi_c \sigma_t + A_2^i A_2^i \sigma_t^2 \right\}
\end{array} \right.
\]

Again, this is a non-linear system in three equations and three unknowns, which we solve numerically.

The equity risk premium on dividend claim \( i \) is:

\[
E_t \left[ r_{t+1}^i \right] = \left[ \varphi_d^i + \frac{\alpha - \rho}{1 - \rho} \left\{ A_1^i A_1^i \varphi_c^2 + \chi A_1^i A_2^i \varphi_c \sigma_t + \chi A_2^i A_1^i \varphi_c \sigma_t + A_2^i A_2^i \sigma_t^2 \right\} \right] \sigma_t^2.
\]
Correlations expected dividend growth and expected return Expected stock returns and expected dividend growth rates on any portfolio $i$ are equal to:

$$
E_{i} \left[ r_{t+1}^{i} \right] = r_0^{i} + \rho x_t^{i} + \left[ \phi_{d,s}^{i} - A_2^{i} (\kappa_t^{i} - \nu_t^{i}) \right] (\sigma_t^{2} - \bar{\sigma}^2),
$$

$$
E_{i} \left[ \Delta d_{t+1}^{i} \right] = \mu_t^{i} + \phi_{d,s}^{i} x_t^{i} + \phi_{d,s}^{i} (\sigma_t^{2} - \bar{\sigma}^2).
$$

The unconditional variance of these quantities is:

$$
V \left[ E_{i} \left[ r_{t+1}^{i} \right] \right] = \rho^2 V[x_t^{i}] + [\phi_{d,s}^{i} - A_2^{i} (\kappa_t^{i} - \nu_t^{i})]^2 V[\sigma_t^{2} - \bar{\sigma}^2] + 2\rho [\phi_{d,s}^{i} - A_2^{i} (\kappa_t^{i} - \nu_t^{i})] \text{Cov}[x_t^{i}, \sigma_t^{2} - \bar{\sigma}^2],
$$

$$
V \left[ E_{i} \left[ \Delta d_{t+1}^{i} \right] \right] = (\phi_{d,s}^{i})^2 V[x_t^{i}] + (\phi_{d,s}^{i})^2 V[\sigma_t^{2} - \bar{\sigma}^2] + 2\phi_{d,s}^{i} \phi_{d,s}^{i} \text{Cov}[x_t^{i}, \sigma_t^{2} - \bar{\sigma}^2].
$$

The unconditional covariance between expected stock returns and expected dividend growth rates is

$$
\text{Cov} \left( E_{i} \left[ r_{t+1}^{i} \right], E_{i} \left[ \Delta d_{t+1}^{i} \right] \right) = \phi_{d,s}^{i} \rho V[x_t^{i}] + \phi_{d,s}^{i} [\phi_{d,s}^{i} - A_2^{i} (\kappa_t^{i} - \nu_t^{i})] V[\sigma_t^{2} - \bar{\sigma}^2] + (\rho \phi_{d,s}^{i} + \phi_{d,s}^{i} [\phi_{d,s}^{i} - A_2^{i} (\kappa_t^{i} - \nu_t^{i})]) \text{Cov}[x_t^{i}, \sigma_t^{2} - \bar{\sigma}^2].
$$

This then leads to the unconditional correlation, which is straightforward to compute.

The important thing to note is that this covariance is (1) asset-specific because it depends on both $\phi_{d,s}^{i}$ and $\phi_{d,s}^{i}$, and (2) it can be positive or negative depending on $\phi_{d,s}^{i}$ and $\phi_{d,s}^{i}$, holding fixed $\chi$. In the standard Bansal-Yaron model, the covariance between the state variables is zero and $\phi_{d,s}^{i} = 0$. In that case, the covariance between expected returns and expected dividend growth is essentially equal to $\chi$. Hence, its sign depends on the sign of $\chi$, and it is not stock-specific. The previous section showed a high positive correlation between expected returns and expected dividend growth for growth stocks and a negative correlation for value stocks. Our model is able to generate this.

The unconditional correlation between expected and unexpected dividend growth is zero for all portfolios by the law of iterated expectations:

$$
\text{Corr} \left( E_{i} \left[ \Delta d_{t+1}^{i} \right], \Delta d_{t+1}^{i} - E_{i} \left[ \Delta d_{t+1}^{i} \right] \right) = \text{Corr}(\phi_{d,s}^{i} x_t^{i} + \phi_{d,s}^{i} (\sigma_t^{2} - \bar{\sigma}^2), \phi_{d,s}^{i} \sigma_t^{1} u_{t+1}) = 0.
$$

Likewise, the correlation between expected and unexpected returns is zero. This is the same as in the standard BY model.

B.3 Bond Pricing

The Risk-free Rate According to equation (29), the expression for the risk-free rate is given by

$$
y_t^{1} = -A(1) - B(1)x_t - C(1)(\sigma_t^{2} - \bar{\sigma}^2).
$$

$$
A(1) = m_0 + .5\Gamma(0)\bar{\sigma}^2, 
$$

$$
B(1) = -\rho, 
$$

$$
C(1) = \frac{\alpha - \rho}{1 - \rho}(\kappa_t^{1} - \nu_t^{1})A_2 + .5\Gamma(0),
$$

42
where the notation $\Gamma(0)$ is defined as:

$$\Gamma(0) \equiv \alpha^2 + \left(\frac{\alpha - \rho}{1 - \rho}\right)^2 \{ (A_1^c \varphi_c)^2 + (A_2^c \sigma_w)^2 + 2 \chi A_1^c \varphi_c A_2^c \sigma_w \}$$  \hspace{1cm} (44)$$

**The Real Yield Curve** The price of a $\tau$-period real zero-coupon bond satisfies:

$$P_t(\tau) = E_t \left[ e^{m_{t+1} + \log P_{t+1}(\tau - 1)} \right].$$

This defines a recursion with $P_t(0) = 1$. The corresponding bond yield is $y_t(\tau) = -\log(P_t(\tau))/\tau$. It is easy to show that the LRR model gives rise to an affine real term structure (Bansal and Shaliastovich 2007):

$$y_t(\tau) = \frac{A(\tau)}{\tau} - \frac{B(\tau)}{\tau} x_t - \frac{C(\tau)}{\tau} (\sigma_t^2 - \bar{\sigma}^2),$$

The coefficients $A(\tau), B(\tau),$ and $C(\tau)$ satisfy the following recursions:

$$A(\tau + 1) = m_0 + A(\tau) + .5 \Gamma(\tau) \bar{\sigma}^2,$$

$$B(\tau + 1) = \rho_x B(\tau) - \rho,$$

$$C(\tau + 1) = \nu_1 C(\tau) + \frac{\alpha - \rho}{1 - \rho} (\kappa_1^c - \nu_1) A_2^c + .5 \Gamma(\tau)$$  \hspace{1cm} (47)$$

where

$$\Gamma(\tau) \equiv \alpha^2 + \left[ B(\tau) - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \right]^2 \varphi_c^2 + \left[ C(\tau) - \frac{\alpha - \rho}{1 - \rho} A_2^c \right]^2 \sigma_w^2$$

$$+ 2 \chi \left[ B(\tau) - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \right] \varphi_c \left[ C(\tau) - \frac{\alpha - \rho}{1 - \rho} A_2^c \right] \sigma_w,$$  \hspace{1cm} (48)$$

and where the recursion is initialized at $A(0) = 0, B(0) = 0,$ and $C(0) = 0.$

*Proof.* Guess and verify

$$P_t(\tau + 1) = E_t \left[ \exp \{ m_{t+1} + \log \{ P_{t+1}(\tau) \} \} \right]$$

$$= E_t \left[ \exp \{ m_{t+1} + A(\tau) + B(\tau)x_{t+1} + C(\tau)(\sigma_{t+1}^2 - \bar{\sigma}^2) \} \right]$$

$$= \exp \{ m_0 + A(\tau) + (B(\tau)\rho_x - \rho) x_t + \nu_1 C(\tau) + \frac{\alpha - \rho}{1 - \rho} (\kappa_1^c - \nu_1) A_2^c \} \left( \sigma_t^2 - \bar{\sigma}^2 \right) \times$$

$$E_t \left[ \exp \left\{ -\alpha \sigma_t \eta_{t+1} + \left[ B(\tau) - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \right] \varphi_c \sigma_t \eta_{t+1} + \left[ C(\tau) - \frac{\alpha - \rho}{1 - \rho} A_2^c \right] \sigma_w \sigma_t \eta_{t+1} \right\} \right].$$

Note that the conditional expectation term is equal to $\exp \{ .5 \Gamma(\tau) \bar{\sigma}^2 \}$, where $\Gamma(\tau)$ is defined in (48). The rest follows from recognizing that

$$\log P_t(\tau + 1) = A(\tau + 1) + B(\tau + 1)x_t + C(\tau + 1)(\sigma_t^2 - \bar{\sigma}^2)$$

and matching up coefficients. \( \square \)

**Real Bond Returns** Define the 1-period log return on a real bond of maturity $\tau$ as:

$$r_{t+1}^p(\tau) = \tau y_t(\tau) - (\tau - 1) y_{t+1}(\tau - 1).$$
The real bond risk premium (the expected excess return with Jensen adjustment) is:

$$r_{t+1}^b - y_t(1) = \tau y_t(\tau) - (\tau - 1)y_{t+1}(\tau - 1) - y_t(1).$$

In general, the excess log return on buying an $\tau$-period real bond at time $t$ and selling it at time $t + m$ as an $\tau - m$ period bond is:

$$\tau y_t(\tau) - (\tau - m)y_{t+m}(\tau - m) - my_t(1).$$

The same expression holds for nominal excess bond returns, which we denote with a $^\$ superscript.

Innovations in one-period real bond returns are given by

$$r_{t+1}^b - E_t[r_{t+1}^b] = (\tau - 1) (-y_{t+1}(\tau - 1) + E_t[y_{t+1}(\tau - 1)]) = B(\tau - 1)\varphi_\tau \sigma_{t+1} + C(\tau - 1)\sigma_w \sigma_{t+1}. $$

The conditional variance of the bond return therefore equals:

$$V_t[r_{t+1}^b(\tau)] = \left\{ [B(\tau - 1)\varphi_\tau]^2 + [C(\tau - 1)\sigma_w]^2 + 2\chi B(\tau - 1)\varphi_\tau C(\tau - 1)\sigma_w \right\} \sigma_t^2.$$ 

The real bond risk premium (the expected excess return with Jensen adjustment) is:

$$E_t[r_{t+1}^b(\tau)] = \tau y_t(\tau) - (\tau - 1)E_t[y_{t+1}(\tau - 1)] - y_t(1) + .5V_t\left[r_{t+1}^b(\tau)\right],$$

$$= [.5\Gamma(0) - .5\Gamma(\tau - 1) + .5 \left\{ [B(\tau - 1)\varphi_\tau]^2 + [C(\tau - 1)\sigma_w]^2 + 2\chi B(\tau - 1)\varphi_\tau C(\tau - 1)\sigma_w \right\} \sigma_t^2, $$

$$\left\{ B(\tau - 1)\frac{\alpha - \rho}{\kappa_1 - \rho_x} \varphi_\tau^2 + C(\tau - 1)\frac{\alpha - \rho}{1 - \rho} A_2^2 \sigma_w^2 $$

$$+ \chi B(\tau - 1)\varphi_\tau \frac{\alpha - \rho}{1 - \rho} A_2^2 \sigma_w + \chi C(\tau - 1)\varphi_\tau \frac{\alpha - \rho}{\kappa_1 - \rho_x} \sigma_w \right\} \sigma_t^2.$$ 

We have verified that this risk premium equals $-\text{Cov}_t \left[m_{t+1}, r_{t+1}^b(\tau) - E_t\left[r_{t+1}^b(\tau)\right]\right].$

**The Nominal Yield Curve** The price of a $\tau$-period nominal zero-coupon bond satisfies:

$$P_t^N(\tau) = E_t \left[e^{m_{t+1} + \log P_{t+1}^N(\tau - 1)}\right].$$

This defines a recursion with $P_t^N(0) = 1$. The corresponding bond yield is $y_t^N(\tau) = -\log(P_t^N(\tau))/\tau$. Nominal bond yields are also affine in the state vector (Bansal and Shaliastovich 2007):

$$y_t^N(\tau) = -\frac{A^N(\tau)}{\tau} - \frac{B^N(\tau)}{\tau} x_t - \frac{C^N(\tau)}{\tau} (\sigma_t^2 - \bar{\sigma}^2) - \frac{D^N(\tau)}{\tau} (\bar{\sigma}_t - \mu_x).$$

The coefficients $A(\tau), B(\tau), C(\tau), D(\tau)$ satisfy the following recursions

$$A^N(\tau + 1) = m_0 - \mu_x + A^N(\tau) + .5 \left[-\sigma_x + D^N(\tau)\sigma_x\right]^2 + .5\Gamma^N(\tau)\bar{\sigma}^2;$$

$$B^N(\tau + 1) = B^N(\tau)p_x - \rho + D^N(\tau)\phi_\pi;$$

$$C^N(\tau + 1) = C^N(\tau)\nu_1 + \frac{\alpha - \rho}{1 - \rho} (\kappa_1 - \nu_1) A_2^T + .5\Gamma^N(\tau)\bar{\sigma};$$

$$D^N(\tau + 1) = D^N(\tau)p_\pi - 1;$$

44
where bond measures the inflation risk premium at the short end of the term structure. It is given by

\[ \text{Inflation Risk Premium} = \text{coefficients} \]

We conjecture that the \( \phi \) takes the same sign for all periods. When in principle, the inflation risk premium can be either negative or positive depending on the sign of the coefficients in the process of verifying this conjecture using the Euler equation:

\[
\begin{align*}
P_t^s(t + 1) &= E_t[\exp(m_t^s + \log(P_{t+1}^s(\tau)))] \\
&= E_t[\exp(m^{t+1} - \pi^{t+1} + A^s(\tau) + B^s(\tau)x_t + C^s(\tau)(\pi^{t+1} - \sigma^2) + D^s(\tau)(\bar{\pi}_t + \mu^\tau))] \\
e&\exp(m_0 - \pi^\tau + A^s(\tau) + B^s(\tau)\rho_x - \rho + D^s(\tau)\phi^\tau)x_t + \left[C^s(\tau)\mu_t + \frac{\alpha - \rho}{1 - \rho}(\kappa_t^x - \mu_t)x_t \right] + \left[D^s(\tau)\rho_x - 1 + (\bar{\pi}_t - \mu^\tau)\right] \times \\
e&\exp\left\{ -\alpha - \varphi_{\pi, \eta} + D^s(\tau)\zeta_{\pi, \eta} \right\} \sigma_t q_{t+1} \times \\
&\left\{ -\frac{\alpha - \rho}{1 - \rho} - \varphi_{\pi, e} + B^s(\tau) + D^s(\tau)\zeta_{\pi, e} \right\} \varphi_e \sigma_t e_{t+1} \times \\
&\left\{ -\frac{\alpha - \rho}{1 - \rho} - A^s(\tau) + C^s(\tau) \right\} \sigma_w \sigma_t u_{t+1} + \left[-\sigma^\pi + D^s(\tau)\sigma_z \right] \xi_{t+1} \right\}
\end{align*}
\]

We use the joint standard normality of the four mutually-orthogonal shocks. Taking logs and collecting terms, we obtain a linear equation for \( \log(P_t^s(t + 1)) \) where the coefficients \( A^s(\tau + 1) \) through \( D^s(\tau + 1) \) satisfy \( 49 \)– \( 52 \).

We have verified that the one-period nominal short rate satisfies:

\[
y_t^s(1) = -E_t[m_t^s - .5V_t[m_t^s]] = -E_t[m_t^s - \pi_t^s] - .5V_t[m_t^s - \pi_t^s]
\]

for the coefficients \( A^s(\tau) \) through \( D^s(\tau) \) in \( 49 \)– \( 52 \).

**Inflation Risk Premium** The expected excess return on a one-period nominal bond over a one-period real bond measures the inflation risk premium at the short end of the term structure. It is given by \( -\text{cov}_t[m_t^s, \pi_t^s] \), where

\[
\text{Inflation Risk Premium} = \alpha \varphi_{\pi, \eta} + \frac{\alpha - \rho}{\kappa_t^\pi - \rho_x} \varphi_{\pi, e} \varphi_e^2 + \chi \frac{\alpha - \rho}{1 - \rho} A^s \sigma_w \varphi_{\pi, e} \varphi_e \sigma_t^2.
\]

In principle, the inflation risk premium can be either negative or positive depending on the sign of the \( \chi \). However, it takes the same sign for all periods. When \( \varphi_{\pi, \eta} = 0 \) and \( \chi = 0 \), which is the case in (Bansal and Shaliastovich 2007), the inflation risk premium equals the second term.

**Nominal Bond Returns** Define the 1-period log return on a nominal bond of maturity \( \tau \) as:

\[
i_{t+1}^b(\tau) = \tau y_{t+1}^s(\tau) - (\tau - 1)y_{t+1}^s(\tau - 1),
\]

\[
= A^s(\tau) - B^s(\tau)x_t - C^s(\tau)(\sigma_t^2 - \sigma^2) - D^s(\tau)(\bar{\pi}_t - \mu^\pi)
\]

\[
+ A^s(\tau - 1) + B^s(\tau - 1)x_{t+1} + C^s(\tau - 1)(\sigma_{t+1}^2 - \sigma^2) + D^s(\tau - 1)(\bar{\pi}_{t+1} - \mu^\pi)
\]

45
Expected nominal bond returns are:

\[ E_t \left[ r^{b,s}_{t+1}(\tau) \right] = A^s(\tau - 1) - A^s(\tau) + \left[ B^3(\tau - 1)\rho_\pi + D^3(\tau - 1)\phi_\pi - B^s(\tau) \right] x_t \]

\[ + \left[ C^s(\tau - 1)\nu_t - C^s(\tau) \right] (\sigma_t^2 - \bar{\sigma}^2) + \left[ D^3(\tau - 1)\rho_\pi - D^s(\tau) \right] (\tilde{\pi}_t - \mu_\pi). \]

Innovation in nominal bond returns are:

\[ r^{b,s}_{t+1}(\tau) - E_t \left[ r^{b,s}_{t+1}(\tau) \right] = (\tau - 1) \left( -g^s_{t+1}(\tau - 1) + E_t \left[ g^s_{t+1}(\tau - 1) \right] \right), \]

\[ = D^s(\tau - 1)\zeta_{\pi,\eta}\sigma_t\eta_{t+1} + \left[ B^3(\tau - 1) + D^3(\tau - 1)\zeta_{\pi,\eta} \right] \varphi_c\sigma_t e_{t+1} \]

\[ + C^s(\tau - 1)\sigma_u\sigma_t w_{t+1} + D^3(\tau - 1)\sigma_u \xi_{t+1}. \]

Expected excess bond returns adjusted for a Jensen term, or nominal bond risk premia, are given by:

\[ E_t \left[ r^{b,c,s}_{t+1}(\tau) \right] = -Cov_t \left[ m^c_{t+1}, r^{b,c,s}_{t+1}(\tau) \right] \]

\[ = Cov_t \left[ \pi_{t+1}, r^{b,c,s}_{t+1}(\tau) \right] - Cov_t \left[ m_{t+1}, r^{b,c,s}_{t+1}(\tau) \right] \]

\[ = r^c_{0}(\tau) + F^c(\tau)(\sigma_t^2 - \bar{\sigma}^2) \]

where

\[ r^c_{0}(\tau) = D^c(\tau - 1)\sigma_u \sigma_t + F^c(\tau)\bar{\sigma}^2 \]

\[ F^c(\tau) = (\varphi_{\pi,\eta} + \alpha) D^c(\tau - 1)\zeta_{\pi,\eta} + \left( \varphi_{\pi,\eta} + \frac{\alpha - \rho}{\kappa_1 - \rho_\pi} \left[ B^c(\tau - 1) + D^c(\tau - 1)\zeta_{\pi,\eta} \right] \varphi_c + \frac{\alpha - \rho}{1 - \rho} A_2^c C^c(\tau - 1)\sigma_w^2 + \right. \]

\[ \left. \chi \frac{\alpha - \rho}{1 - \rho} A_2^c \sigma_w \left[ B^c(\tau - 1) + D^c(\tau - 1)\zeta_{\pi,\eta} \right] \varphi_c + \chi \frac{\alpha - \rho}{\kappa_1 - \rho_\pi} \varphi_c C^c(\tau - 1)\sigma_w \right) \]

We have used the fact that:

\[ Cov_t \left[ \pi_{t+1}, r x^s_{t+1}(\tau) \right] = \left\{ \varphi_{\pi,\eta} D^c(\tau - 1)\zeta_{\pi,\eta} + \varphi_{\pi,\pi} \left[ B^c(\tau - 1) + D^c(\tau - 1)\zeta_{\pi,\eta} \right] \varphi_c \right. \]

\[ + \left. \chi \varphi_{\pi,\eta} \varphi_c C^c(\tau - 1)\sigma_w \right\} \sigma_t^2 + D^c(\tau - 1)\sigma_u \sigma_t \]

\[ Cov_t \left[ m_{t+1}, r x^s_{t+1}(\tau) \right] = \left\{ -\alpha D^c(\tau - 1)\zeta_{\pi,\eta} - \frac{\alpha - \rho}{\kappa_1 - \rho_\pi} \left[ B^c(\tau - 1) + D^c(\tau - 1)\zeta_{\pi,\eta} \right] \varphi_c \right. \]

\[ - \left. \frac{\alpha - \rho}{1 - \rho} A_2^c C^c(\tau - 1)\sigma_w^2 - \chi \frac{\alpha - \rho}{1 - \rho} A_2^c \sigma_w \left[ B^c(\tau - 1) + D^c(\tau - 1)\zeta_{\pi,\eta} \right] \varphi_c \right. \]

\[ - \left. \chi \frac{\alpha - \rho}{\kappa_1 - \rho_\pi} \varphi_c C^c(\tau - 1)\sigma_w \right\} \sigma_t^2. \]

In sum, nominal expected excess bond returns (nominal bond risk premia) only vary over time because economic uncertainty \((\sigma_t^2 - \bar{\sigma}^2)\) varies. The loading \(F(t)\) measures their sensitivity to this factor; it depends on the maturity of the bond.
The Cochrane-Piazzesi Factor Define the nominal forward rate for a loan between period \( t + \tau \) and \( t + \tau + 1 \) as usual from nominal bond prices:

\[
f_t^\delta(\tau) = \log P_t^\delta(\tau - 1) - \log P_t^\delta(\tau).
\]

In the model, the forward rate is

\[
f_t^\delta(\tau) = (A^\delta(\tau - 1) - A^\delta(\tau)) + \left( B^\delta(\tau - 1) - B^\delta(\tau) \right) x_t + \
(C^\delta(\tau - 1) - C^\delta(\tau)) (\sigma_t^2 - \bar{\sigma}^2) + \left( D^\delta(\tau - 1) - D^\delta(\tau) \right) (\bar{\sigma}_t - \mu_\tau)
\]

Given that the nominal model has three state variables, we can invert the state variables from any three forward rates, say \( f_t^\delta(1), f_t^\delta(3), \) and \( f_t^\delta(5) \). Put differently, one linear combination of forward rates is perfectly correlated with \((\sigma_t^2 - \bar{\sigma}^2)\). This is the linear combination of interest, because this state variable is the only source of time variation in bond and stock risk premia. Hence, this linear combination is the best predictor of future bond returns. It is perfectly correlated with expected bond returns. We call in the Cochrane-Piazzesi factor because the weights \((c_1, c_3, c_5)\) display a tent-shaped function as in Cochrane and Piazzesi (2005):

\[
CP_t = c_1 f_t^\delta(1) + c_3 f_t^\delta(3) + c_5 f_t^\delta(5) = b_{CP}(\sigma_t^2 - \bar{\sigma}^2).
\]

The main text explains how \( b_{CP} \) is pinned down.

B.4 Relationship between stocks and bonds

The Equity Risk Premium and the Campbell-Shiller Decomposition Expected discounted future equity returns and dividend growth rates are given by:

\[
r_{\delta t}^{i,H} = E_t \left[ \sum_{j=1}^{\infty} (\kappa_t^i)^{-j} r_{t+j}^{i} \right] = \frac{r_0}{\kappa_t^i - 1} + \frac{\rho}{\kappa_t^i - \rho_x} x_t + \left[ \frac{\phi_t^{d,\sigma} - A_2^\delta (\kappa_t^i - \nu_1)}{\kappa_t^i - \nu_1} \right] (\sigma_t^2 - \bar{\sigma}^2)
\]

\[
\Delta d_t^{i,H} = E_t \left[ \sum_{j=1}^{\infty} (\kappa_t^i)^{-j} \Delta d_{t+j}^{i} \right] = \frac{\mu_{\delta t}^{d}}{\kappa_t^i - 1} + \frac{\phi_t^{d,\sigma}}{\kappa_t^i - \rho_x} x_t + \left[ \frac{\phi_t^{d,\sigma}}{\kappa_t^i - \nu_1} \right] (\sigma_t^2 - \bar{\sigma}^2)
\]

From these expressions, it is easy to see that

\[
pd_{t}^{i} = \frac{\kappa_t^0}{\kappa_t^i - 1} + \Delta d_{t}^{i,H} - r_{t}^{i,H},
\]

and to compute the elements of the variance-decomposition:

\[
V[pd_{t}^{i}] = Cov[pd_{t}^{i}, \Delta d_{t}^{i,H}] + Cov[pd_{t}^{i}, -r_{t}^{i,H}] = V[\Delta d_{t}^{i,H}] + V[r_{t}^{i,H}] - 2Cov[\Delta d_{t}^{i,H}, r_{t}^{i,H}].
\]

We can also derive an expression for the expected future real bond yield (the risk-free rate):

\[
y_t^{H}(1) = E_t \left[ \sum_{j=1}^{\infty} (\kappa_t^i)^{-j} y_{t+j-1}(1) \right] = -\frac{A(1)}{\kappa_t^i - 1} + \frac{\rho}{\kappa_t^i - \rho_x} x_t + \frac{C(1)}{\kappa_t^i - \nu_1} (\sigma_t^2 - \bar{\sigma}^2)
\]
This makes it clear that the first term in the expected future stock return expression is a risk-free rate effect. It drops out once we look at expected excess returns:

\[ r_{i,t}^{i.e,H} = r_{i,t}^{i,H} - y_{H}^{i}(1) = r_{i,0}^{i} + A(1) + \left[ \frac{\phi_{d,\sigma}^i - A_{2}^i(\kappa_1^i - \nu_1) + C(1)}{\kappa_1^i - \nu_1} \right] (\sigma_t^2 - \bar{\sigma}^2) \]

The covariance between expected future dividend growth and expected future returns:

\[ Cov \left( r_{i,t}^{i,H}, \Delta d_{i,t}^{i,H} \right) = \frac{\phi_{d,\sigma}^i \phi_{d,x}^i}{(\kappa_1^i - \rho_x)^2} V[x_t] + \frac{\phi_{d,\sigma}^i \phi_{d,x}^i}{(\kappa_1^i - \nu_1)^2} V[\sigma_t^2 - \bar{\sigma}^2] \]

\[ + \frac{\rho \phi_{d,\sigma}^i \phi_{d,x}^i A_{2}^i(\kappa_1^i - \nu_1) + C(1)}{(\kappa_1^i - \rho_x)(\kappa_1^i - \nu_1)} Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]. \]

It is similar to that of expected one-period ahead dividend growth and expected returns in equation (54). Indeed, if \( \rho_x = \nu_1 \), then it equals the latter covariance divided by \( (\kappa_1^i - \rho_x)^2 = (\kappa_1^i - \nu_1)^2 \). Note that when \( \chi = 0 \) and \( \phi_{d,\sigma}^i = 0 \), only the first term survives in the covariance between expected returns and expected growth rates. Because \( \rho > 0 \) and \( \phi_{d,x}^i > 0 \), it is always positive. Furthermore, this is a pure risk-free rate effect: the correlation between the expected excess return and expected dividend growth rate is exactly zero when \( \chi = 0 \) and \( \phi_{d,\sigma}^i = 0 \). In our model, this is no longer necessarily the case because of the additional cash-flow effect coming through \( \phi_{d,\sigma}^i \neq 0 \):

\[ Cov \left( r_{i,t}^{i,e,H}, \Delta d_{i,t}^{i,H} \right) = \frac{\phi_{d,\sigma}^i \phi_{d,x}^i - A_{2}^i(\kappa_1^i - \nu_1) + C(1)}{(\kappa_1^i - \nu_1)^2} V[\sigma_t^2 - \bar{\sigma}^2] \]

\[ + \frac{\rho \phi_{d,\sigma}^i \phi_{d,x}^i A_{2}^i(\kappa_1^i - \nu_1) + C(1)}{(\kappa_1^i - \rho_x)(\kappa_1^i - \nu_1)} Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]. \]

Finally, for the consumption claim, we get the following expected future return, excess return, and growth expressions:

\[ r_{i,t}^{c,H} = \frac{r_{i,0}^{c}}{\kappa_1^c - 1} + \frac{\rho}{\kappa_1^c - \rho_x} x_t - A_{2}^c(\sigma_t^2 - \bar{\sigma}^2) \] (58)

\[ \Delta c_{i,t}^{H} = \frac{\mu_0}{\kappa_1^c - 1} + \frac{1}{\kappa_1^c - \rho_x} x_t \] (59)

\[ r_{i,t}^{c.e,H} = \frac{r_{i,0}^{c} - A(1)}{\kappa_1^c - 1} + \frac{1}{\kappa_1^c - \rho_x} A_{2}^c(\kappa_1^c - \nu - 1) + .5 \bar{\Gamma}(0) (\sigma_t^2 - \bar{\sigma}^2) \] (60)

**Correlation stock returns and dividend growth with lagged CP** We recall the expressions for expected returns and expected dividend growth rates on stock portfolios in equation (51)–(52). It follows that the unconditional lagged CP-betas of expected returns, \( \tilde{\beta}_{CP}^{r} \equiv \beta (r_{i+1}^{i}, CP_{t}) \), and expected excess returns \( \tilde{\beta}_{CP}^{r,e} \equiv \beta (r_{i+1}^{i} - y_{t}(1), CP_{t}) \) are:

\[ \tilde{\beta}_{CP}^{r} = \frac{\rho}{b_{CP}} \frac{Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} + \frac{\phi_{d,\sigma}^i - A_{2}^i(\kappa_1^i - \nu_1)}{b_{CP}}. \] (61)

\[ \tilde{\beta}_{CP}^{r,e} = \frac{\phi_{d,\sigma}^i - A_{2}^i(\kappa_1^i - \nu_1) + \frac{\alpha_{C}}{b_{CP}} A_{2}^c(\kappa_1^c - \nu - 1) + .5 \bar{\Gamma}(0)}{b_{CP}}. \] (62)

The unconditional lagged CP-beta for expected returns may or may not be positive. The unconditional lagged CP-beta for realized returns is the same as for expected returns: \( \beta (r_{i+1}^{i} - y_{t}(1), CP_{t}) \). The reason is that unexpected returns have a zero unconditional correlation with \( CP_{t} \) by the law of iterated expectations.

48
The unconditional correlation between realized dividend growth rates $\Delta d_{t+1}$ and $CP_t$ is the same as the correlation with expected growth rates because unexpected dividend growth rates have a zero correlation with $CP_t$, because $\text{Cov} (\varphi_d^2 \sigma_t u_{t+1}, \sigma_t^2 - \bar{\sigma}^2) = 0$, again because of the law of iterated expectations.

$$\beta^d_{CP} \equiv \beta (\Delta d_{t+1}, CP_t) = \beta \left( E_t [\Delta d_{t+1}], CP_t \right) = \frac{\phi_{d,x}^t \text{Cov}[x_t, \sigma_t^2 - \bar{\sigma}^2]}{b_{CP}} + \frac{\phi_{d,\sigma}^t}{b_{CP}}.$$ \hspace{1cm} (63)

**Correlation stock returns and dividend growth with contemporaneous CP** It is straightforward to derive the unconditional contemporaneous CP-betas of returns $\tilde{\beta}^d_{CP} \equiv \beta (r_{t+1}^d, CP_{t+1})$ and dividend growth rates $\tilde{\beta}^d_{CP} \equiv \beta (\Delta d_{t+1}, CP_{t+1})$ are:

$$\tilde{\beta}^d_{CP} \equiv \nu_1 \beta^d_{CP} + \left( \chi A_1^t \frac{\varphi_e}{\sigma_w} + A_2^t \right) \frac{(1 - \nu_1^2)}{b_{CP}}.$$ \hspace{1cm} (64)

$$\tilde{\beta}^c_{CP} \equiv \nu_1 \beta^c_{CP} + \left( \chi A_1^t \frac{\varphi_w}{\sigma_w} + A_2^t \right) \frac{(1 - \nu_1^2)}{b_{CP}}.$$ \hspace{1cm} (65)

$$\tilde{\beta}^d_{CP} \equiv \nu_1 \tilde{\beta}^d_{CP}.$$ \hspace{1cm} (66)

Note that the contemporaneous CP-betas for dividend growth are proportional to the lagged CP-betas, with a constant of proportionality of $\nu_1$. The reason is that dividend growth innovations are orthogonal to CP innovations because $u_{t+1}$ and $w_{t+1}$ are uncorrelated. The contemporaneous CP-beta of returns is also multiplied by $\nu_1$, but there is an additional term $A_2^t (1 - \nu_1^2)$ added, which arises from the fact that $pd$ innovations and CP innovations are both driven by the shock $w_{t+1}$ and from the fact that innovations to $w_{t+1}$ and $e_{t+1}$ may be correlated ($\chi \neq 0$). The contemporaneous CP beta of excess returns is the same as the one of returns.

**Correlation nominal bond returns with lagged CP** For comparison, we also work out the betas of nominal bond returns with respect to lagged CP, $\tilde{\beta}^b_{CP}(\tau) \equiv \beta \left( r_{t+1}^b(\tau), CP_t \right)$, and of excess nominal bond returns $\tilde{\beta}^{b,e}_{CP}(\tau) \equiv \beta \left( r_{t+1}^{b,e}(\tau) - y_t^e(1), CP_t \right)$:

$$\tilde{\beta}^b_{CP}(\tau) = \frac{\rho \frac{\text{Cov}[x_t, \sigma_t^2 - \bar{\sigma}^2]}{b_{CP}} + \frac{1}{b_{CP}} \frac{\text{Cov}[\bar{\pi}_t - \mu_\pi, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} - \frac{-\alpha - \rho (\kappa_1^e - \nu_1) A_2^e - 5 \Gamma^b(\tau - 1)}{b_{CP}}}{\frac{5 \Gamma^b(0) - 5 \Gamma^b(\tau - 1)}{b_{CP}}}.$$ \hspace{1cm} (67)

$$\tilde{\beta}^{b,e}_{CP}(\tau) = \frac{-\alpha - \rho (\kappa_1^e - \nu_1) A_2^e - 5 \Gamma^b(\tau - 1)}{b_{CP}}.$$ \hspace{1cm} (68)

The first equation follows because $\tilde{\beta} \left( r_{t+1}^b(\tau), CP_t \right) = \beta \left( E_t \left[ r_{t+1}^{b,e}(\tau) \right], CP_t \right)$. Note that the first term is common with the beta of stock returns; it is a yield curve effect. The second term is also a yield curve effect. Both disappear if we look at excess returns.

**Correlation nominal bond returns with contemporaneous CP** The contemporaneous CP beta of nominal bond returns, $\tilde{\beta}^b_{CP}(\tau) \equiv \beta \left( r_{t+1}^b(\tau), CP_{t+1} \right)$, and excess returns are given by:

$$\tilde{\beta}^b_{CP}(\tau) = \nu_1 \tilde{\beta}^b_{CP}(\tau) + \left( \chi B_1^b(\tau - 1) + D_1^b(\tau - 1) \zeta_{\pi,e} \right) \frac{\varphi_e}{\sigma_w} + C^b(\tau - 1) \frac{(1 - \nu_1^2)}{b_{CP}}.$$ \hspace{1cm} (69)

$$\tilde{\beta}^{b,e}_{CP}(\tau) = \nu_1 \tilde{\beta}^{b,e}_{CP}(\tau) + \left( \chi B_1^b(\tau - 1) + D_1^b(\tau - 1) \zeta_{\pi,e} \right) \frac{\varphi_e}{\sigma_w} + C^b(\tau - 1) \frac{(1 - \nu_1^2)}{b_{CP}}.$$ \hspace{1cm} (70)
**Beta decomposition**  Recall the definition of a return

\[ r_{i+1}^t = \kappa_0^i + \Delta d_{i+1}^t + pd_{i+1}^t - \kappa_1^i pd_i^t \]

and the definition of the \( pd \) ratio:

\[ pd_i^t = A_0^i + A_1^i x_t + A_2^i (\sigma_t^2 - \bar{\sigma}^2) \]

Therefore, the contemporaneous return beta is:

\[ \hat{\beta}_{CP}^r - \hat{\beta}_{CP}^d = -\frac{A_1^i (\kappa_1^i \nu_1 - 1) \text{Cov}[x_t, \sigma_t^2 - \bar{\sigma}^2]}{b_{CP}} - \frac{A_2^i (\kappa_1^i \nu_1 - 1)}{b_{CP}} \]

If \( \kappa_1^i \nu_1 - 1 > 0 \), which is a statement about the relative persistence of \( x \) and \( \sigma_t^2 - \bar{\sigma}^2 \), and \( \chi < 0 \), then both terms on the RHS are positive because \( A_1^i > 0 \) and \( A_2^i < 0 \). Since value stocks have higher \( A_1^i \) and more negative \( A_2^i \), this explains the increasing pattern in the LHS from growth to value.

Another way to understand this is by using

\[ pd_i^t = \frac{\kappa_0^i}{\kappa_1^i - 1} + \Delta d_{i+1}^t - r_{i+1}^t \]

Therefore, the contemporaneous return beta is:

\[ \hat{\beta}_{CP}^r - \hat{\beta}_{CP}^d = \beta \left[ CP_{t+1}, \Delta d_{i+1}^t - \kappa_1^i \Delta d_i^t \right] - \beta \left[ CP_{t+1}, r_{i+1}^t - \kappa_1^i r_i^t \right] \tag{71} \]

The first beta on the RHS measures the covariance of innovations of \( CP_{t+1} \) with \( \text{news} \) about future dividend growth rates. It equals:

\[ \beta \left[ CP_{t+1}, \Delta d_{i+1}^t - \kappa_1^i \Delta d_i^t \right] = \frac{-\phi_{d,x}^i \nu_1 \text{Cov}[x_t, \sigma_t^2 - \bar{\sigma}^2]}{b_{CP}} - \frac{\phi_{d,\sigma}^i \nu_1}{b_{CP}} \]
\[ + \left\{ \chi \frac{\phi_{d,x}^i}{(\kappa_1^i - \rho_x) \sigma_w} + \frac{\phi_{d,\sigma}^i}{(\kappa_1^i - \nu_1)} \right\} (1 - \nu_1^2) \tag{72} \]

The second beta on the RHS measures the covariance of innovations of \( CP_{t+1} \) with \( \text{news} \) about future stock returns. It equals:

\[ \beta \left[ CP_{t+1}, r_{i+1}^t - \kappa_1^i r_i^t \right] = \frac{-\rho \nu_1 \text{Cov}[x_t, \sigma_t^2 - \bar{\sigma}^2]}{b_{CP}} - \frac{\phi_{d,\sigma}^i - A_2^i (\kappa_1^i - \nu_1)}{b_{CP}} \]
\[ + \left\{ \chi \frac{\rho}{(\kappa_1^i - \rho_x) \sigma_w} + \frac{\phi_{d,\sigma}^i - A_2^i (\kappa_1^i - \nu_1)}{\kappa_1^i - \nu_1} \right\} (1 - \nu_1^2) \tag{73} \]

It is easy to verify that the difference between these two betas delivers the first expression we derived for \( \hat{\beta}_{CP}^r - \hat{\beta}_{CP}^d \).
B.5 The Term Structure of Equity

We derive the price of dividend strips (or zero-coupon equity), the price of a claim to unit of dividends of firm $i$ at some future date $t$. Recall the log-linearized stock return expression for security or portfolio $i$:

$$r_{i,t+1}^i = \kappa_0^i + \Delta d_{i,t+1}^i + pd_{i,t+1}^i - \kappa_1^i pd_{i,t}^i. \quad (76)$$

Log price-dividend ratios on dividend strips of horizon $\tau$, $\tilde{pd}_{i}^i(\tau)$, are affine in the state vector:

$$\tilde{pd}_{i}^i(\tau) = A^m(\tau) + B^m(\tau)x_t + C^m(\tau)(\sigma_t^2 - \bar{\sigma}^2),$$

where the coefficients $A^m(\tau)$, $B^m(\tau)$, and $C^m(\tau)$ follow recursions

$$A^m(\tau + 1) = A^m(\tau) + m_0 + \mu_\delta + \frac{1}{2} \Gamma^m(\tau)\bar{\sigma}^2, \quad B^m(\tau + 1)' = \rho_z B^m(\tau) + \phi_{d,x}^i - \rho,$$

$$C^m(\tau + 1)' = \nu_1 C^m(\tau) + \phi_{d,\sigma}^i + \frac{\alpha - \rho}{1 - \rho}(\kappa_i^\delta - \nu_1)A_2^\delta + \frac{1}{2} \Gamma^m(\tau), \quad \Gamma^m(\tau) = \alpha^2 + \varphi_d^2 - 2\alpha \varphi_d t + \left[B^m(\tau) - \frac{\alpha - \rho}{1 - \rho} A_1^\delta \right]^2 \varphi_e^2 + \left[C^m(\tau) - \frac{\alpha - \rho}{1 - \rho} A_2^\delta \right] \psi_c \sigma_w + 2\chi \left[B^m(\tau) - \frac{\alpha - \rho}{1 - \rho} A_1^\delta \right] \left[C^m(\tau) - \frac{\alpha - \rho}{1 - \rho} A_2^\delta \right] \psi_c \sigma_w$$

initialized at $A^m(0) = 0$, $B^m(0) = 0$, and $C^m(0) = 0$.

\textit{Proof.} We conjecture that the log $t + 1$-price of a $\tau$-period strip, scaled by the dividend in period $t + 1$, is affine in the state

$$\tilde{pd}_{i,t+1}(\tau) = A^m(\tau) + B^m(\tau)'x_{t+1} + C^m(\tau)(\sigma_{t+1}^2 - \bar{\sigma}^2)$$

and solve for the coefficients $A^m(\tau+1)$, $B^m(\tau+1)$, and $B^m(\tau+1)$ in the process of verifying this conjecture using the Euler equation:

$$\exp\tilde{pd}_{i}^i(\tau + 1) = E_t \left[ \exp\{m_{t+1} + \Delta d_{i,t+1}^i + \tilde{pd}_{i}^i(\tau)\} \right] = \exp\left[m_0 + \mu_\delta + A^m(\tau) + \phi_{d,x}^i - \rho + B^m(\tau)\rho_x x_t + \left[\phi_{d,\sigma}^i + \frac{\alpha - \rho}{1 - \rho}(\kappa_i^\delta - \nu_1)A_2^\delta + C^m(\tau)\nu_1\right](\sigma_t^2 - \bar{\sigma}^2)\right] \times E_t \left[ \exp\{-\alpha \sigma_t \eta_{t+1} + \left[B^m(\tau) - \frac{\alpha - \rho}{1 - \rho} A_1^\delta \right] \varphi_e \sigma_t e_{t+1} + \left[C^m(\tau) - \frac{\alpha - \rho}{1 - \rho} A_2^\delta \right] \sigma_w \sigma_t w_{t+1} + \varphi_d \sigma_t u_{t+1} \right]$$

We use the log-normality of the shocks to work out the expectation. Taking logs and collecting terms, we obtain the conjectured linear expression for $pd_{i}^i(\tau + 1)$ where the recursions are as given above.

\hfill \Box

C Quarterly Calibration LRR Model

The Bansal-Yaron model is calibrated and parameterized to monthly data. Since we want to use data on quarterly consumption and dividend growth, and a quarterly series for the wealth-consumption ratio, we recast the model at quarterly frequencies. We assume that the quarterly process for consumption growth, dividend growth, the low frequency component and the variance has the exact same structure than at the monthly frequency, with mean zero, standard deviation 1 innovations, but with different parameters. This appendix explains how the monthly
parameters map into quarterly parameters. We denote all variables, shocks, and all parameters of the quarterly system with a tilde superscript. Our parameter values are listed at the end of this section, together with details on the simulation approach.

**Preference Parameters** Obviously, the preference parameters do not depend on the horizon (\( \tilde{\alpha} = \alpha \) and \( \tilde{\rho} = \rho \)), except for the time discount factor \( \tilde{\beta} = \beta^3 \). Also, the long-run average log wealth-consumption ratio at the quarterly frequency is lower than at the monthly frequency by approximately \( \log(3) \), because log of quarterly consumption is the log of three times monthly consumption.

**Cash-flow Parameters** We accomplish this by matching the conditional and unconditional mean and variance of log consumption and dividend growth. Log quarterly consumption growth is the sum of log consumption growth of three consecutive months. We obtain \( \Delta \tilde{c}_{t+1} \equiv \Delta c_{t+3} + \Delta c_{t+2} + \Delta c_{t+1} \)

\[
\Delta \tilde{c}_{t+1} = 3\mu_c + (1 + \rho_x + \rho_x^2)x_t + \sigma_t\eta_{t+1} + \sigma_{t+1}\eta_{t+2} + \sigma_{t+2}\eta_{t+3} + (1 + \rho_x)\varphi_c\sigma_t\epsilon_{t+1} + \varphi_c\sigma_{t+1}\epsilon_{t+2} \tag{77}
\]

Log quarterly dividend growth looks similar:

\[
\Delta \tilde{d}_{t+1} = 3\mu_d + \phi(1 + \rho_x + \rho_x^2)x_t + \varphi_d\sigma_t u_{t+1} + \varphi_d\sigma_{t+1} u_{t+2} + \varphi_d\sigma_{t+2} u_{t+3} + \phi(1 + \rho_x)\varphi_c\sigma_t \epsilon_{t+1} + \phi\varphi_c\sigma_{t+1} \epsilon_{t+2} \tag{78}
\]

First, we rescale the long-run component in the quarterly system, so that the coefficient on it in the consumption growth equation is still 1:

\[ \tilde{x}_t = (1 + \rho_x + \rho_x^2)x_t. \]

Second, we equate the unconditional mean of consumption and dividend growth:

\[ \tilde{\mu} = 3\mu, \quad \tilde{\mu}_d = 3\mu_d. \]

These imply that we also match the the conditional mean of consumption growth:

\[ E_t[\Delta c_{t+3} + \Delta c_{t+2} + \Delta c_{t+1}] = 3\mu + (1 + \rho_x + \rho_x^2)x_t = \tilde{\mu} + \tilde{x}_t = E_t[\Delta \tilde{c}_{t+1}] \]

Third, we also match the conditional mean of dividend growth by setting the quarterly leverage parameter \( \tilde{\phi} = \phi \).

Fourth, we match the unconditional variance of quarterly consumption growth:

\[
V[\Delta \tilde{c}_{t+1}] = (1 + \rho_x + \rho_x^2)^2V[x_t] + \sigma^2 [3 + (1 + \rho_x)^2\varphi_c^2 + \varphi_c^2]
= (1 + \rho_x + \rho_x^2)^2\varphi_c^2\sigma_c^2 + \sigma^2 [3 + (1 + \rho_x)^2\varphi_c^2 + \varphi_c^2]
= \frac{\varphi_c^2\sigma_c^2}{1 - \rho_x^2} + \sigma^2
\]

The first and second equalities use the law of iterated expectations to show that

\[
V[\sigma_{t+j}\eta_{t+j+1}] = E [E_{t+j} \{\sigma_{t+j}\eta_{t+j+1}\}] - (E [E_{t+j} \{\sigma_{t+j}\eta_{t+j+1}\}])^2 = E [\sigma_{t+j}^2] - 0 = \sigma^2
\]

and the same argument applies to terms of type \( V[\sigma_{t+j}\epsilon_{t+j+1}] \). Coefficient matching on the variance of consumption
expression delivers expressions for $\tilde{\sigma}^2$ and $\tilde{\varphi}_e$:

$$\tilde{\sigma}^2 = \sigma^2 \left[ 3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2 \right]$$

$$\tilde{\varphi}_e^2 = \frac{\varphi_e^2 (1 - \rho_x^2) (1 + \rho_x + \rho_x^2)^2 \sigma^2}{1 - \rho_x^2} \tilde{\sigma}^2$$

$$= \frac{(1 - \rho_x^6) (1 + \rho_x + \rho_x^2)^2}{1 - \rho_x^2} \frac{\varphi_e^2}{3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2},$$

where the third equality uses the first equality. Note that we imposed $\tilde{\rho}_x = \rho_x^3$, which follows from a desire to match the persistence of the long-run cash-flow component. Recursively substituting, we find that the three-month ahead $x$ process has the following relationship to the current value:

$$x_{t+3} = \rho_x^3 x_t + \varphi_x \sigma_{t+2} e_{t+3} + \rho_x \varphi_x \sigma_{t+1} e_{t+2} + \rho_x^2 \varphi_x \sigma_t e_{t+1}$$

which compares to the quarterly equation

$$\tilde{x}_{t+1} = \tilde{\rho}_x \tilde{x}_t + \tilde{\varphi}_x \tilde{\sigma}_t \tilde{e}_{t+1}$$

The two processes now have the same auto-correlation and unconditional variance.

Fifth, we match the unconditional variance of dividend growth. Given the assumptions we have made sofar, this pins down $\varphi_d$:

$$\tilde{\varphi}_d^2 = \frac{3 \varphi_d^2 + \phi^2 (1 + \rho_x)^2 \varphi_e^2 + \phi^2 \varphi_e^2}{3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2} \tilde{\sigma}_d^2$$

Sixth, we match the autocorrelation and the unconditional variance of economic uncertainty $\sigma_t^2$. Iterating forward, we obtain an expression that relates variance in month $t$ to the one in month $t + 3$:

$$\sigma_{t+3}^2 - \sigma^2 = \nu_1^3 (\sigma_t^2 - \sigma^2) + \sigma_w \nu_1^2 w_{t+1} + \sigma_w \nu_1^1 w_{t+2} + \sigma_w w_{t+3}$$

By setting $\tilde{\nu}_1 = \nu_1^3$ and $\tilde{\sigma}_w = \sigma_w^2 (1 + \nu_1^2 + \nu_1^4)$, we match the autocorrelation and variance of the quarterly equation

$$\tilde{\sigma}_{t+1}^2 - \sigma^2 = \tilde{\nu}_1 (\tilde{\sigma}_t^2 - \sigma^2) + \tilde{\sigma}_w \tilde{w}_{t+1}$$
Table 1: Market Prices of Risk in the Data

This table reports slope and intercept estimates from the second-stage estimation of average returns on the 10 value portfolios (top panel) or the 25 size- and value portfolios (bottom panel) on the bond risk factor exposures (from the first-stage estimation). The intercept is in percent per year. The bond risk factor is the Cochrane-Piazzesi CP factor in the left panel and the 20-l-quarter yield spread in the right panel. The intercept is expressed as a percent per year. The last four rows denote the mean absolute pricing error (MAPE) and the root mean squared pricing error (RMSE) across the portfolios, also expressed as a percent per year. We also report the cross-sectional $R^2$ from the second stage estimation, and the sample length from the first-stage estimation. The different columns denote different sampling frequencies.

### 10 Value Portfolios (1952-2007)

<table>
<thead>
<tr>
<th>predictor:</th>
<th>CP factor</th>
<th>Yield Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M$</td>
<td>$Q$</td>
</tr>
<tr>
<td>slope</td>
<td>1.57</td>
<td>2.57</td>
</tr>
<tr>
<td>$[t - \text{stat}]$</td>
<td>5.31</td>
<td>4.56</td>
</tr>
<tr>
<td>intercept</td>
<td>6.49</td>
<td>5.81</td>
</tr>
<tr>
<td>$[t - \text{stat}]$</td>
<td>14.85</td>
<td>9.15</td>
</tr>
<tr>
<td>MAPE</td>
<td>0.65</td>
<td>0.75</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.75</td>
<td>0.85</td>
</tr>
<tr>
<td>$R^2$</td>
<td>77.9</td>
<td>72.2</td>
</tr>
<tr>
<td>$T$</td>
<td>666</td>
<td>222</td>
</tr>
</tbody>
</table>

### 25 Size- and Value Portfolios (1952-2007)

<table>
<thead>
<tr>
<th>predictor:</th>
<th>CP factor</th>
<th>Yield Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M$</td>
<td>$Q$</td>
</tr>
<tr>
<td>slope</td>
<td>2.20</td>
<td>2.81</td>
</tr>
<tr>
<td>$[t - \text{stat}]$</td>
<td>10.19</td>
<td>10.26</td>
</tr>
<tr>
<td>intercept</td>
<td>5.86</td>
<td>5.67</td>
</tr>
<tr>
<td>$[t - \text{stat}]$</td>
<td>14.85</td>
<td>13.22</td>
</tr>
<tr>
<td>MAPE</td>
<td>0.80</td>
<td>0.84</td>
</tr>
<tr>
<td>RMSE</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>$R^2$</td>
<td>81.9</td>
<td>82.1</td>
</tr>
<tr>
<td>$T$</td>
<td>666</td>
<td>222</td>
</tr>
</tbody>
</table>
Table 2: Cross-Sectional Equity Return Analysis: Subsamples

This table reports slope and intercept estimates from the second-stage estimation of average returns on the 10 value portfolios (top panel) or the 25 size- and value portfolios (bottom panel) on the bond risk factor exposures from the first-stage estimation. The bond risk factor is the Cochrane-Piazzesi \( CP \) factor. In the left panel, the \( CP \) factor is not re-estimated (same as in the full sample), whereas in the right panel it is re-estimated on the sub-sample in question.

### 10 Book-to-market Portfolios (Monthly Frequency)

<table>
<thead>
<tr>
<th></th>
<th>CP from full sample</th>
<th>CP estimated on subsample</th>
</tr>
</thead>
<tbody>
<tr>
<td>slope</td>
<td>2.37 1.67 0.75</td>
<td>2.96 1.43 0.70</td>
</tr>
<tr>
<td></td>
<td>[6.91] [3.31] [2.43]</td>
<td>[7.42] [3.01] [1.98]</td>
</tr>
<tr>
<td>intercept</td>
<td>4.43 3.75 11.39</td>
<td>5.40 3.12 11.58</td>
</tr>
<tr>
<td></td>
<td>[8.59] [2.78] [11.38]</td>
<td>[14.41] [2.04] [8.86]</td>
</tr>
<tr>
<td>MAPE</td>
<td>0.60 1.24 0.70</td>
<td>0.56 1.34 0.74</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.73 1.47 0.76</td>
<td>0.68 1.54 0.81</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>85.6 57.7 42.4</td>
<td>87.3 53.1 33.0</td>
</tr>
<tr>
<td>( T )</td>
<td>540 402 264</td>
<td>540 402 264</td>
</tr>
</tbody>
</table>

### 25 Size- and Value Portfolios (Monthly Frequency)

<table>
<thead>
<tr>
<th></th>
<th>CP from full sample</th>
<th>CP estimated on subsample</th>
</tr>
</thead>
<tbody>
<tr>
<td>slope</td>
<td>2.59 2.09 1.29</td>
<td>3.14 1.73 1.42</td>
</tr>
<tr>
<td></td>
<td>[13.15] [5.41] [5.47]</td>
<td>[10.87] [4.65] [6.94]</td>
</tr>
<tr>
<td>intercept</td>
<td>4.35 3.44 12.28</td>
<td>5.88 3.58 14.66</td>
</tr>
<tr>
<td></td>
<td>[11.48] [3.01] [18.20]</td>
<td>[17.17] [2.77] [17.19]</td>
</tr>
<tr>
<td>MAPE</td>
<td>0.78 1.46 1.16</td>
<td>0.92 1.58 1.27</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.94 1.83 1.70</td>
<td>1.10 1.98 1.47</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>88.3 56.0 56.6</td>
<td>83.7 48.4 67.7</td>
</tr>
<tr>
<td>( T )</td>
<td>540 402 264</td>
<td>540 402 264</td>
</tr>
</tbody>
</table>
Table 3: Cross-Sectional Equity Return Analysis: Comparison with 3- and 4-Factor Model

This table reports slope and intercept estimates from the second-stage estimation of average returns on the 25 size- and value portfolios on the factor exposures from the first-stage estimation. The factors are the 3 Fama-French factors (first panel), the 4 Carhart factors (second panel), the yield spread and the 3 Fama-French factors (third panel), and the yield spread and the 4 Fama-French factors (fourth panel).

<table>
<thead>
<tr>
<th>25 Size- and Value Portfolios (1952-2007)</th>
<th>CP</th>
<th>yspr</th>
<th>3-f</th>
<th>4-f</th>
<th>CP+3-f</th>
<th>yspr+3-f</th>
<th>CP+4-f</th>
<th>yspr+4-f</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP</td>
<td>2.20</td>
<td></td>
<td>1.56</td>
<td></td>
<td>1.32</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[10.19]</td>
<td></td>
<td>[3.77]</td>
<td></td>
<td>[2.97]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yspr</td>
<td>1.31</td>
<td></td>
<td>1.35</td>
<td></td>
<td>1.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[5.43]</td>
<td></td>
<td>[7.42]</td>
<td></td>
<td>[6.68]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mkt</td>
<td>-11.97</td>
<td>-0.35</td>
<td>-6.30</td>
<td>-1.02</td>
<td>-1.01</td>
<td>2.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-2.77]</td>
<td>[-0.06]</td>
<td>[-1.69]</td>
<td>[-0.38]</td>
<td>[-0.19]</td>
<td>[0.78]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SMB</td>
<td>1.76</td>
<td>1.45</td>
<td>1.75</td>
<td>1.99</td>
<td>1.59</td>
<td>1.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[3.17]</td>
<td>[2.79]</td>
<td>[4.03]</td>
<td>[6.77]</td>
<td>[3.59]</td>
<td>[6.21]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HML</td>
<td>4.57</td>
<td>5.12</td>
<td>4.41</td>
<td>4.67</td>
<td>4.73</td>
<td>4.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[6.72]</td>
<td>[7.74]</td>
<td>[8.24]</td>
<td>[13.02]</td>
<td>[8.20]</td>
<td>[13.11]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UMD</td>
<td>40.31</td>
<td></td>
<td>20.06</td>
<td>13.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2.32]</td>
<td></td>
<td>[1.24]</td>
<td>[1.32]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>intercept</td>
<td>5.86</td>
<td>1.96</td>
<td>19.25</td>
<td>8.29</td>
<td>13.72</td>
<td>8.42</td>
<td>8.76</td>
<td>4.84</td>
</tr>
<tr>
<td></td>
<td>[14.85]</td>
<td>[1.42]</td>
<td>[4.28]</td>
<td>[1.33]</td>
<td>[3.58]</td>
<td>[3.03]</td>
<td>[1.65]</td>
<td>[1.37]</td>
</tr>
<tr>
<td>MAPE</td>
<td>0.80</td>
<td>1.30</td>
<td>0.91</td>
<td>0.86</td>
<td>0.69</td>
<td>0.52</td>
<td>0.69</td>
<td>0.47</td>
</tr>
<tr>
<td>RMSE</td>
<td>1.04</td>
<td>1.62</td>
<td>1.20</td>
<td>1.07</td>
<td>0.92</td>
<td>0.62</td>
<td>0.88</td>
<td>0.58</td>
</tr>
<tr>
<td>$R^2$</td>
<td>81.9</td>
<td>56.2</td>
<td>75.9</td>
<td>81.0</td>
<td>85.8</td>
<td>93.6</td>
<td>87.0</td>
<td>94.4</td>
</tr>
</tbody>
</table>
Table 4: Contemporaneous Betas of Bond Portfolios in the Data

This table reports slope and intercept estimates from the second-stage estimation of average returns on the 5 CRSP bond portfolio returns on the level factor betas from a first-stage estimation. The intercept is in percent per year. The level factor is the 5-year Fama-Bliss yield in the left panel and the first principal components of the 1- through 5-year Fama-Bliss yields in the right panel. The last four rows denote the mean absolute pricing error (MAPE) and the root mean squared pricing error (RMSE) across the 25 portfolios. Both are expressed in percent per year. We also report the cross-sectional $R^2$ from the second stage estimation, and the sample length from the first-stage estimation. The different columns denote different sampling frequencies.

<table>
<thead>
<tr>
<th>Predictor</th>
<th>5-Year Bond Yield factor</th>
<th>Level Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>Q</td>
</tr>
<tr>
<td>slope</td>
<td>-1.06</td>
<td>-0.89</td>
</tr>
<tr>
<td>[t − stat]</td>
<td>[-3.05]</td>
<td>[-3.04]</td>
</tr>
<tr>
<td>intercept</td>
<td>1.14</td>
<td>0.78</td>
</tr>
<tr>
<td>[t − stat]</td>
<td>[10.43]</td>
<td>[5.69]</td>
</tr>
<tr>
<td>MAPE</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td>$R^2$</td>
<td>75.7</td>
<td>75.5</td>
</tr>
</tbody>
</table>

| T           | 666 | 222 | 55 | 666 | 222 | 55 |
Table 5: Linear Predictability in the Data

This table reports slope and intercept estimates from a predictability regression of cumulative (discounted) BM10-BM1 excess returns (top panel) or the cumulative discounted aggregate excess stock market return (bottom panel) on the bond risk premium. The bond risk factor is the Cochrane-Piazzesi CP factor in the left panel and the 20-1-quarter yield spread in the right panel. The first column denotes the holding period over which returns are cumulated. The first 59 months are discarded so that all returns are computed over the same sample. All cumulative results are expressed on a per annum basis so that the slope coefficients have comparable magnitudes across rows. The full sample is monthly data from June 1952 until December 2007. T-stats are computed using Newey-West standard errors with $k - 1$ lags, where $k$ is the horizon in months.

<table>
<thead>
<tr>
<th>predictor:</th>
<th>Value-Growth Portfolio Return</th>
<th></th>
<th></th>
<th>Aggregate Stock Market Return</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>slope</td>
<td>t-stat</td>
<td>$R^2$</td>
<td>slope</td>
<td>t-stat</td>
<td>$R^2$</td>
</tr>
<tr>
<td>1-qtr</td>
<td>1.810</td>
<td>1.596</td>
<td>0.96</td>
<td>-0.485</td>
<td>-0.218</td>
<td>0.02</td>
</tr>
<tr>
<td>2-qtr</td>
<td>1.480</td>
<td>1.350</td>
<td>1.18</td>
<td>-0.152</td>
<td>-0.069</td>
<td>0.00</td>
</tr>
<tr>
<td>3-qtr</td>
<td>1.613</td>
<td>1.542</td>
<td>2.04</td>
<td>0.942</td>
<td>0.419</td>
<td>0.23</td>
</tr>
<tr>
<td>1-yr</td>
<td>1.790</td>
<td>1.784</td>
<td>3.30</td>
<td>1.913</td>
<td>0.867</td>
<td>1.25</td>
</tr>
<tr>
<td>2-yr</td>
<td>1.543</td>
<td>2.141</td>
<td>5.03</td>
<td>1.760</td>
<td>1.118</td>
<td>2.18</td>
</tr>
<tr>
<td>3-yr</td>
<td>0.951</td>
<td>1.550</td>
<td>3.20</td>
<td>-0.069</td>
<td>-0.057</td>
<td>0.01</td>
</tr>
<tr>
<td>4-yr</td>
<td>0.692</td>
<td>1.151</td>
<td>2.38</td>
<td>-0.756</td>
<td>-0.743</td>
<td>0.94</td>
</tr>
<tr>
<td>5-yr</td>
<td>0.378</td>
<td>0.673</td>
<td>0.94</td>
<td>-0.876</td>
<td>-1.099</td>
<td>1.69</td>
</tr>
</tbody>
</table>
Table 6: Non-Linear Predictability in the Data

This table reports slope and intercept estimates from a predictability regression of cumulative (discounted) BM10-BM1 excess returns (top panel) or the cumulative discounted aggregate excess stock market return (bottom panel) on a non-linear function of the bond risk premium. The non-linear function of the bond risk premium is constructed as $CP_t > 0.160$ in the left panel and $yspr_t > 0.168$ in the right panel. The first column denotes the holding period over which returns are cumulated. The first 59 months are discarded so that all returns are computed over the same sample. All cumulative results are expressed on a per annum basis so that the slope coefficients have comparable magnitudes across rows. The full sample is monthly data from June 1952 until December 2007. T-stats are computed using Newey-West standard errors with $k-1$ lags, where $k$ is the horizon in months.

### Value-Growth Portfolio Return

<table>
<thead>
<tr>
<th>predictor:</th>
<th>CP factor</th>
<th>Yield Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>slope</td>
<td>t-stat</td>
</tr>
<tr>
<td>1-qtr</td>
<td>3.119</td>
<td>2.446</td>
</tr>
<tr>
<td>2-qtr</td>
<td>2.244</td>
<td>2.028</td>
</tr>
<tr>
<td>3-qtr</td>
<td>2.232</td>
<td>2.076</td>
</tr>
<tr>
<td>1-yr</td>
<td>2.496</td>
<td>2.242</td>
</tr>
<tr>
<td>2-yr</td>
<td>2.228</td>
<td>2.370</td>
</tr>
<tr>
<td>3-yr</td>
<td>1.521</td>
<td>2.424</td>
</tr>
<tr>
<td>4-yr</td>
<td>0.956</td>
<td>1.859</td>
</tr>
<tr>
<td>5-yr</td>
<td>0.427</td>
<td>0.884</td>
</tr>
</tbody>
</table>

### Aggregate Stock Market Return

<table>
<thead>
<tr>
<th>predictor:</th>
<th>CP factor</th>
<th>Yield Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>slope</td>
<td>t-stat</td>
</tr>
<tr>
<td>1-qtr</td>
<td>-0.271</td>
<td>-0.227</td>
</tr>
<tr>
<td>2-qtr</td>
<td>0.046</td>
<td>0.040</td>
</tr>
<tr>
<td>3-qtr</td>
<td>0.555</td>
<td>0.547</td>
</tr>
<tr>
<td>1-yr</td>
<td>1.031</td>
<td>1.237</td>
</tr>
<tr>
<td>2-yr</td>
<td>0.941</td>
<td>1.467</td>
</tr>
<tr>
<td>3-yr</td>
<td>0.582</td>
<td>1.276</td>
</tr>
<tr>
<td>4-yr</td>
<td>0.737</td>
<td>2.070</td>
</tr>
<tr>
<td>5-yr</td>
<td>0.807</td>
<td>2.288</td>
</tr>
</tbody>
</table>
Table 7: Unified SDF Model for Stocks and Bonds - Pricing Errors

Panel A of this table reports pricing errors on 10 book-to-market sorted stock portfolios, the value-weighted market portfolio, and five bond portfolios of maturities 1, 2, 5, 7, and 10 years. They are expressed in percent per year. Each column corresponds to a different SDF model, as described in the text, apart from the second and third column. In that case, we only use the set of test assets to estimate the same stochastic discount factor. The main text contains further details. The last row reports the mean absolute pricing error across all 11 securities (MAPE). Panel B reports the estimates of the prices of risk. In all specifications, we set $\Lambda_1 = 0$. The data are monthly for 1952.6 until 2007.12.

<table>
<thead>
<tr>
<th></th>
<th>SDF 1</th>
<th>SDF 2</th>
<th>SDF 2 (bonds only)</th>
<th>SDF 3</th>
<th>SDF 4</th>
<th>SDF 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>BM1</td>
<td>5.8888</td>
<td>5.3241</td>
<td>5.8256</td>
<td>-3.4608</td>
<td>0.0729</td>
<td>0.0239</td>
</tr>
<tr>
<td>BM2</td>
<td>6.4994</td>
<td>3.3423</td>
<td>5.8862</td>
<td>-2.1722</td>
<td>-0.6638</td>
<td>-0.3818</td>
</tr>
<tr>
<td>BM3</td>
<td>7.3279</td>
<td>2.7917</td>
<td>6.4327</td>
<td>-1.2192</td>
<td>-0.9517</td>
<td>-0.7492</td>
</tr>
<tr>
<td>BM4</td>
<td>7.2821</td>
<td>3.7721</td>
<td>6.5913</td>
<td>-0.8931</td>
<td>0.1355</td>
<td>0.1568</td>
</tr>
<tr>
<td>BM5</td>
<td>8.1376</td>
<td>3.7695</td>
<td>7.2660</td>
<td>0.5395</td>
<td>0.5638</td>
<td>0.6731</td>
</tr>
<tr>
<td>BM6</td>
<td>8.4560</td>
<td>3.7300</td>
<td>7.5011</td>
<td>0.7664</td>
<td>0.9151</td>
<td>0.6941</td>
</tr>
<tr>
<td>BM7</td>
<td>8.3899</td>
<td>0.9870</td>
<td>6.8493</td>
<td>1.0857</td>
<td>-1.2690</td>
<td>-1.5589</td>
</tr>
<tr>
<td>BM8</td>
<td>10.0592</td>
<td>3.5705</td>
<td>8.7182</td>
<td>2.6343</td>
<td>0.9732</td>
<td>0.8755</td>
</tr>
<tr>
<td>BM9</td>
<td>10.3731</td>
<td>3.2154</td>
<td>8.8965</td>
<td>2.6478</td>
<td>0.3608</td>
<td>0.0556</td>
</tr>
<tr>
<td>BM10</td>
<td>11.1598</td>
<td>4.8792</td>
<td>9.8865</td>
<td>2.7126</td>
<td>0.7499</td>
<td>1.0304</td>
</tr>
<tr>
<td>Market</td>
<td>6.8726</td>
<td>3.2714</td>
<td>6.1601</td>
<td>-1.6128</td>
<td>-0.7461</td>
<td>-0.7520</td>
</tr>
<tr>
<td>1-yr</td>
<td>1.0830</td>
<td>-2.3229</td>
<td>0.3626</td>
<td>0.9190</td>
<td>-0.6594</td>
<td>-0.3207</td>
</tr>
<tr>
<td>2-yr</td>
<td>1.2438</td>
<td>-3.7555</td>
<td>0.1878</td>
<td>0.9974</td>
<td>-0.7408</td>
<td>-0.3147</td>
</tr>
<tr>
<td>5-yr</td>
<td>1.5251</td>
<td>-5.8999</td>
<td>-0.0417</td>
<td>1.1422</td>
<td>-0.2558</td>
<td>-0.2039</td>
</tr>
<tr>
<td>7-yr</td>
<td>1.7474</td>
<td>-6.0792</td>
<td>0.0952</td>
<td>1.2163</td>
<td>0.3209</td>
<td>0.2558</td>
</tr>
<tr>
<td>10-yr</td>
<td>1.3764</td>
<td>-6.6745</td>
<td>-0.3240</td>
<td>0.6856</td>
<td>0.3625</td>
<td>0.1921</td>
</tr>
<tr>
<td>MAPE</td>
<td>6.0889</td>
<td>3.9616</td>
<td>5.0641</td>
<td>1.5441</td>
<td>0.6088</td>
<td>0.5149</td>
</tr>
</tbody>
</table>

Panel B: Estimates of Prices of Risk ($\Lambda_0$)

<table>
<thead>
<tr>
<th></th>
<th>SDF 1</th>
<th>SDF 2</th>
<th>SDF 2 (bonds only)</th>
<th>SDF 3</th>
<th>SDF 4</th>
<th>SDF 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP factor</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>58.53</td>
<td>83.72</td>
</tr>
<tr>
<td>(Modified) Level factor</td>
<td>0</td>
<td>-67.96</td>
<td>-14.30</td>
<td>0</td>
<td>-21.85</td>
<td>-53.59</td>
</tr>
<tr>
<td>(Modified) Slope factor</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(Modified) Curvature factor</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Aggregate stock market return</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4.07</td>
<td>3.12</td>
<td>3.65</td>
</tr>
</tbody>
</table>
Table 8: Quarterly Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Our model</th>
<th>BS 2008</th>
<th>BY 2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>8</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>$\rho$</td>
<td>2/3</td>
<td>2/3</td>
<td>2/3</td>
</tr>
<tr>
<td>$\beta$</td>
<td>.9961</td>
<td>.9961</td>
<td>.9970</td>
</tr>
<tr>
<td>Consumption Process</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>.0048</td>
<td>.0048</td>
<td>.0045</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>.0056</td>
<td>.0056</td>
<td>.0135</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>.9732</td>
<td>.9732</td>
<td>.9383</td>
</tr>
<tr>
<td>$\varphi_e$</td>
<td>.1617</td>
<td>.1617</td>
<td>.1264</td>
</tr>
<tr>
<td>$\nu_l$</td>
<td>EST</td>
<td>.9880</td>
<td>.9615</td>
</tr>
<tr>
<td>$\sigma_{w}$</td>
<td>.1984e-5/\bar{\sigma}</td>
<td>.1984e-5</td>
<td>.3932e-5</td>
</tr>
<tr>
<td>$\chi$</td>
<td>EST</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Dividend Process</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu^m_d$</td>
<td>.0045</td>
<td>.0048</td>
<td>.0045</td>
</tr>
<tr>
<td>$\varphi^m_d$</td>
<td>5.9860</td>
<td>5.9860</td>
<td>4.4960</td>
</tr>
<tr>
<td>$\phi^m_{d,x}$</td>
<td>EST</td>
<td>1.5</td>
<td>3</td>
</tr>
<tr>
<td>$\iota^m$</td>
<td>.10</td>
<td>.10</td>
<td>NA</td>
</tr>
<tr>
<td>$\phi^m_{d,\sigma}$</td>
<td>EST</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Inflation Process</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_\pi$</td>
<td>.0096</td>
<td>.0096</td>
<td>NA</td>
</tr>
<tr>
<td>$\varphi_{\pi,\eta}$</td>
<td>0</td>
<td>0</td>
<td>NA</td>
</tr>
<tr>
<td>$\varphi_{\pi,e}$</td>
<td>-2</td>
<td>-2</td>
<td>NA</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>.0061</td>
<td>.0061</td>
<td>NA</td>
</tr>
<tr>
<td>$\rho_\pi$</td>
<td>.5718</td>
<td>.5718</td>
<td>NA</td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>-35</td>
<td>-35</td>
<td>NA</td>
</tr>
<tr>
<td>$\zeta_{\pi,\eta}$</td>
<td>0</td>
<td>0</td>
<td>NA</td>
</tr>
<tr>
<td>$\zeta_{\pi,e}$</td>
<td>-1</td>
<td>-1</td>
<td>NA</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>6.9282e-6</td>
<td>6.9282e-6</td>
<td>NA</td>
</tr>
</tbody>
</table>

Notes: This Table lists our benchmark parameter choices. All parameters are expressed in quarterly units. See Appendix C on how we go from the monthly parameters of Bansal and Shaliastovich (2007) to the quarterly values in the second column and from the monthly values of Bansal and Yaron (2004) to the quarterly values in the third column.
Table 9: Cash-Flow Parameters and the Value Premium

In Panel A, we use the benchmark calibration of Bansal and Shaliastovich (2007). In panel B, we use our own calibration. The first column reports the unconditional mean dividend growth rate. The second column reports the loading of dividend growth on the long-run risk state variable $x_t$. The third column reports the loading of dividend growth on the economic uncertainty state variable $\sigma^2_t$. The fourth column reports the equity risk premium, the expected excess log stock return (including a Jensen adjustment). The last column reports the mean price-dividend ratio.

**Panel A: B-S Calibration**

<table>
<thead>
<tr>
<th></th>
<th>$\mu^i_d$</th>
<th>$\phi^i_{d,x}$</th>
<th>$\phi^i_{d,\sigma}$</th>
<th>Risk premium</th>
<th>PD</th>
</tr>
</thead>
<tbody>
<tr>
<td>market</td>
<td>1.79</td>
<td>3.52</td>
<td>0</td>
<td>6.82</td>
<td>16.56</td>
</tr>
<tr>
<td>BM1</td>
<td>0.60</td>
<td>3.30</td>
<td>0</td>
<td>6.01</td>
<td>15.13</td>
</tr>
<tr>
<td>BM2</td>
<td>1.65</td>
<td>3.45</td>
<td>0</td>
<td>6.65</td>
<td>16.53</td>
</tr>
<tr>
<td>BM3</td>
<td>2.08</td>
<td>3.70</td>
<td>0</td>
<td>7.24</td>
<td>16.54</td>
</tr>
<tr>
<td>BM4</td>
<td>1.41</td>
<td>3.82</td>
<td>0</td>
<td>7.14</td>
<td>15.07</td>
</tr>
<tr>
<td>BM5</td>
<td>3.16</td>
<td>3.85</td>
<td>0</td>
<td>7.97</td>
<td>18.21</td>
</tr>
<tr>
<td>BM6</td>
<td>2.69</td>
<td>4.09</td>
<td>0</td>
<td>8.14</td>
<td>16.48</td>
</tr>
<tr>
<td>BM7</td>
<td>2.49</td>
<td>4.31</td>
<td>0</td>
<td>8.39</td>
<td>15.57</td>
</tr>
<tr>
<td>BM8</td>
<td>3.17</td>
<td>5.17</td>
<td>0</td>
<td>10.01</td>
<td>14.83</td>
</tr>
<tr>
<td>BM9</td>
<td>3.86</td>
<td>5.05</td>
<td>0</td>
<td>10.20</td>
<td>16.16</td>
</tr>
<tr>
<td>BM10</td>
<td>4.89</td>
<td>5.33</td>
<td>0</td>
<td>11.21</td>
<td>17.39</td>
</tr>
</tbody>
</table>

**Panel B: Own Calibration**

<table>
<thead>
<tr>
<th></th>
<th>$\mu^i_d$</th>
<th>$\phi^i_{d,x}$</th>
<th>$\phi^i_{d,\sigma}$</th>
<th>Risk premium</th>
<th>PD</th>
</tr>
</thead>
<tbody>
<tr>
<td>market</td>
<td>1.79</td>
<td>4.42</td>
<td>-575.17</td>
<td>6.81</td>
<td>17.17</td>
</tr>
<tr>
<td>BM1</td>
<td>0.60</td>
<td>4.17</td>
<td>-536.69</td>
<td>6.09</td>
<td>15.30</td>
</tr>
<tr>
<td>BM2</td>
<td>1.65</td>
<td>4.35</td>
<td>-558.70</td>
<td>6.66</td>
<td>17.05</td>
</tr>
<tr>
<td>BM3</td>
<td>2.08</td>
<td>4.71</td>
<td>-615.94</td>
<td>7.32</td>
<td>17.11</td>
</tr>
<tr>
<td>BM4</td>
<td>1.41</td>
<td>4.74</td>
<td>-614.86</td>
<td>7.13</td>
<td>15.64</td>
</tr>
<tr>
<td>BM5</td>
<td>3.16</td>
<td>4.92</td>
<td>-655.90</td>
<td>8.02</td>
<td>19.31</td>
</tr>
<tr>
<td>BM6</td>
<td>2.69</td>
<td>5.16</td>
<td>-687.54</td>
<td>8.18</td>
<td>17.43</td>
</tr>
<tr>
<td>BM7</td>
<td>2.49</td>
<td>5.39</td>
<td>-716.69</td>
<td>8.43</td>
<td>16.49</td>
</tr>
<tr>
<td>BM9</td>
<td>3.86</td>
<td>6.22</td>
<td>-851.68</td>
<td>10.22</td>
<td>17.96</td>
</tr>
<tr>
<td>BM10</td>
<td>4.89</td>
<td>6.39</td>
<td>-883.53</td>
<td>11.00</td>
<td>20.48</td>
</tr>
</tbody>
</table>
Table 10: Other Moments of Interest

This table reports unconditional means (expressed in percent per year), standard deviations (expressed in percent per year), and the quarterly first-order autocorrelation in the benchmark B-S calibration (first 3 columns) and in our calibration (last 3 columns). The moments reported for stocks are for the market portfolio, and are denoted with a superscript. The notation follows the main text.

<table>
<thead>
<tr>
<th></th>
<th>Panel A: B-S Calibration</th>
<th>Panel B: Our Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>stdev</td>
</tr>
<tr>
<td>( x )</td>
<td>0.00</td>
<td>0.78</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>0.0123</td>
<td>0.0026</td>
</tr>
<tr>
<td>( \bar{\pi} - \mu_{\pi} )</td>
<td>3.84</td>
<td>0.84</td>
</tr>
<tr>
<td>( \Delta c )</td>
<td>1.92</td>
<td>1.36</td>
</tr>
<tr>
<td>( \Delta d^m )</td>
<td>1.79</td>
<td>7.20</td>
</tr>
<tr>
<td>( \pi )</td>
<td>3.84</td>
<td>1.52</td>
</tr>
<tr>
<td>( y(1) )</td>
<td>2.19</td>
<td>0.54</td>
</tr>
<tr>
<td>( y^S(1) )</td>
<td>6.18</td>
<td>0.52</td>
</tr>
<tr>
<td>( y^S(20) )</td>
<td>6.30</td>
<td>0.15</td>
</tr>
<tr>
<td>( y^S(20) - y^S(1) )</td>
<td>0.12</td>
<td>0.73</td>
</tr>
<tr>
<td>( E_t[r^e, \pi_{t+1}(20)] )</td>
<td>-0.94</td>
<td>0.20</td>
</tr>
<tr>
<td>( E_t[r^b, \pi_{t+1}(20)] )</td>
<td>0.21</td>
<td>0.04</td>
</tr>
<tr>
<td>( CP )</td>
<td>0.45</td>
<td>0.09</td>
</tr>
<tr>
<td>( WC )</td>
<td>67.75</td>
<td>5.66</td>
</tr>
<tr>
<td>( PD^m )</td>
<td>16.56</td>
<td>32.77</td>
</tr>
<tr>
<td>( r_{t+1}^m - y_t(1) )</td>
<td>5.68</td>
<td>15.15</td>
</tr>
<tr>
<td>( E_t[r_{t+1}^m - y_t(1)] )</td>
<td>5.68</td>
<td>1.17</td>
</tr>
<tr>
<td>( E_t[\Delta d^m_t] )</td>
<td>1.79</td>
<td>2.75</td>
</tr>
<tr>
<td>( E_t[\pi_{t+1}] )</td>
<td>6.82</td>
<td>1.42</td>
</tr>
</tbody>
</table>
The Cochrane-Piazzesi factor is a linear combination of the one-year nominal yield and 2- through 5-year forward rates. The data run from June 1952 until December 2007.
The figure plots the factor exposures (betas) of the 10 book-to-market portfolio returns $r_{t+1}^{i,c} \equiv r_{t+1}^i - \bar{y}_t(1) + 0.5 \times \text{Var}[r_{t+1}^i]$ to the contemporaneous $CP_{t+1}$ factor. Each bar denotes the slope coefficient of a time-series regression of one portfolio return on the $CP$ factor. The left bar is for growth stocks (log book-to-market), the right bar for value stocks (high book-to-market). The top panel uses monthly return data, the middle panel uses quarterly return data, and the bottom panel uses annual data. The data run from June 1952 until December 2007.
Figure 3: Exposure of 25 Size and Book-to-Market Portfolio Returns to the $CP$ Factor in Data

The figure plots the factor exposures (betas) of the 25 size- and value portfolio returns to the $CP$ factor. Each bar denotes the slope coefficient of a time-series regression of a portfolio return on the $CP$ factor. The first five bars are for the smallest quintile of firms; the last five bars are for the biggest quintile of firms. Within each group of five, the book-to-market ratio increases. The top panel uses monthly return data, the middle panel uses quarterly return data, and the bottom panel uses annual data. The data run from June 1962 until December 2007.
Figure 4: Exposure to the Yield Spread

The figure plots the factor exposures (betas) of the 10 book-to-market portfolio returns (left panels) and the 25 size- and value portfolio returns (right panel) to the yield spread. The data are monthly for June 1952 until December 2007 (top row), quarterly (middle row), and annual (bottom row).
Figure 5: Average Realized vs Predicted Returns: CP Factor

Scatter diagram of average realized versus predicted excess returns. The predicted excess return is generated by the OLS estimation with the CP factor as only explanatory variable. The test assets are the 10 book-to-market portfolio returns (top panel) and the 25 size- and value portfolio returns (bottom panel). The solid line is the 45 degree line. The data run from June 1952 until December 2007.
Figure 6: NBER Recessions, the CP Factor and the Yield Spread

Cochrane-Piazzesi Factor and NBER Recession

Yield spread and NBER Recession
Figure 7: Value versus Growth Returns By CP Factor and Yield Spread Quartiles

The left panels denote average monthly returns on the 10 book-to-market portfolios, multiplied by 1200, by CP quartile. The right panels denote average monthly returns on the 10 book-to-market portfolios, multiplied by 1200, by yield spread quartile. The top row is for the top quartile, the second row for the 50-75 percentile, the third row for the 12-50 percentile, and the bottom row for the lowest 25 percent CP or yield spread realizations.
Figure 8: Exposure of 5 Bond Portfolio Excess Returns to the Level Factor in Data

The figure plots the factor exposures (betas) of five CRSP bond portfolio returns (1-yr, 2-yr, 5-yr, 7-yr, and 10-yr maturities) to the contemporaneous level factor, measured as the 5-year nominal bond yield $y^5(20)$. Each bar denotes the slope coefficient of a time-series regression of one portfolio return on the level factor. The left bar is for the shortest maturity bond, the right bar for longest maturity bonds. The top panel uses monthly return data, the middle panel uses quarterly return data, and the bottom panel uses annual data. The data run from June 1952 until December 2007.

Monthly Frequency

Quarterly Frequency

Annual Frequency
Figure 9: SDF Model-Implied Yields

This figure plots annual yields on nominal bonds of maturities 1- through 5 years as implied by stochastic discount factor model estimated in Section 2.3. The data are Fama-Bliss yields on nominal bonds of maturities 1- through 5 years.
Figure 10: Exposure of 10 Book-to-Market Portfolio Dividend Growth rates to the CP and Yield Spread Factor in Data

The figure plots the factor exposures (betas) of the 10 book-to-market portfolio dividend growth rates $\delta d_{t+1}$ to the contemporaneous $CP_{t+1}$ factor (left panels) and yield spread (right panels). Each bar denotes the slope coefficient of a time-series regression of one portfolio dividend growth rate on the $CP$ factor or the yield spread. The left bar is for growth stocks (log book-to-market), the right bar for value stocks (high book-to-market). The top panel uses monthly return data, the middle panel uses quarterly return data, and the bottom panel uses annual data. The data run from June 1952 until December 2007.
Figure 11: Risk Premia and Betas for $\chi = 0$ and $\phi_{d,\sigma}^i = 0$
Figure 12: Risk Premia and Betas for $\chi > 0$ and $\delta_{d,\sigma}^i < 0$