Dynamic Moral Hazard with
Multiple Correlated Periods and Renegotiation:
Managerial Tenure and Incentives

Florin Şabac *

First version: March 2000
This version: September 29, 2003

Abstract

This paper presents an analysis of dynamic incentives and managerial tenure in an N-period principal-agent model with repeated renegotiation given different time-series properties of the contractible performance measures. The inter-temporal correlation of contracting variables has a significant impact on a manager’s performance over the manager’s tenure. In particular, negatively correlated performance measures lead to increasing performance at the start and declining performance at the end of a manager’s tenure. Moreover, with negatively correlated performance measures, the principal prefers longer managerial tenure, while with positively correlated measures, the opposite holds. In the presence of a fixed switching cost, non-trivial turnover policies exist if performance measures are positively correlated. The results for positive correlation also apply to the career concerns model.

*Department of Accounting and MIS, School of Business, University of Alberta. This paper is based on part of my PhD dissertation at the University of British Columbia. I thank Jerry Feltham, Peter Christensen, Bjørn Jorgensen, and Stefan Reichelstein for their comments and helpful discussions. Thanks also to seminar participants at the University of British Columbia, the University of Alberta, and the Workshop on Accounting and Economics, Madrid 2002.
1 Introduction

The analysis of managerial tenure, turnover, and firm performance surrounding managerial turnover require dynamic agency models with more than two periods. In this paper, I propose a simple $N$-period LEN (Linear contracts, Exponential utility, Normal distributions) model that is sufficiently tractable to allow an examination of dynamic incentives and optimal turnover policies as a function of the characteristics of the information variables used in the manager’s incentive contract. Particular cases include the career concerns model of Gibbons and Murphy [15] and short-term autocorrelated performance measures (for simplicity I refer to this structure as the accounting model), such as accounting earnings.\(^1\) Inter-temporal correlation of the performance measures ensures that the solution to the agency problem is not a simple repetition of the single-period contract, the sufficient conditions of Fudenberg, Holmström, and Milgrom [13] for short-term contracts to replicate long-term contracts are not satisfied.

Although I use full commitment as a benchmark, the main theme is the impact of repeated renegotiation on incentives and managerial tenure. Renegotiation is modelled equivalently by long-term renegotiation-proof contracts as in Fudenberg and Tirole [14] and short-term “fair” contracts that provide both implicit and explicit incentives as in Christensen, Feltham and Šabac [4]. While the effective managerial incentive, and therefore firm performance is not affected by the equivalent contracting mechanisms, the nature of explicit/implicit incentives is sensitive to it: assuming fair contracts gives different explicit/implicit incentives from the career concerns model of Gibbons and Murphy [15]. Demski and Frimor [8] also examine the impact of repeated renegotiation in more than two periods. While they are mainly concerned with “earnings management”, I exogenously assume the agent has no control over the reporting of performance measures.

Not surprisingly, managerial incentives behave differently over a manager’s tenure depending on the informational characteristics of the performance measurement system, in particular their time-series properties. The accounting model with positive correlation produces similar behaviour to

\(^1\)In the case of two periods, the career concerns model is the same as the accounting model and has been extensively used in the literature on performance measurement and incentives, e.g. Meyer [20], Meyer and Vickers [21], Indjejikian and Nanda [17]. The LEN model has been widely used in addressing specific performance measurement and accounting issues, see Dutta and Reichelstein [9, 10], Feltham and Xie [12], and the survey by Lambert [18].
the career concerns model: managerial effort increases through the manager’s tenure. On the other hand, the accounting model with negative correlation gives an inverted U shape for managerial effort during tenure. The behaviour of the accounting model with negative correlation is consistent with findings that CEO performance declines towards retirement, and that the successor CEO’s performance increases immediately following turnover, e.g. Huson, Malatesta, and Parrino [16].

In order to analyze managerial tenure, I assume that commitments to employment duration are possible and that turnover is always anticipated (that is “forced turnover” that depends on random events is exogenously excluded). In this context, the question is: Are there optimal replacement/turnover policies? Or, equivalently, is there an optimal tenure or contract length? The answer, again, depends on the performance measure characteristics. With negatively correlated performance measures, the principal prefers contracting with the agent for the longest duration possible. With positively correlated performance measures, including the career concerns model, there is a range of parameters in which optimal tenure exists, while outside this range corner solutions are optimal (replacing agents every period or never). The evidence that managerial turnover is usually preceded by poor performance is interpreted as support for the view that managerial performance drives turnover, see Huson, Malatesta and Parrino [16] and Murphy and Zimmerman [22]. My results on optimal tenure show that, in some cases, managerial performance and turnover are simultaneously endogenous and may be driven by unobserved commitments to the length of the employment relationship. Moreover, the “horizon problem” examined by Dechow and Sloan [6] is a reflection of the agent’s (anticipated) response to optimally chosen incentives if the horizon is optimally chosen by the principal.

The paper makes three contributions to existing research on dynamic agency. First, it provides a tractable model of N-period agency with correlated performance measures that generalizes Gibbons and Murphy [15] and Christensen, Feltham, and Şabac [5]. Second, the paper provides insights into the link between inter-temporal correlation of performance measures, managerial tenure, and managerial performance. Third, the paper provides results on the impact of performance measure correlation for the choice of optimal managerial tenure/turnover policies.

The remainder of the paper is organized as follows. Section 2 presents the model, Section 3
presents the full commitment benchmark solution, and Section 4 presents the renegotiation-proof contract. In Section 5, I introduce fair contracts, together with the explicit solution to the fair contracts problem. Section 6 discusses dynamic incentives for various information structures. Section 7 provides numerical examples for optimal managerial tenure. Section 8 concludes the paper. Appendix A briefly examines the impact of using a different specification for the agent’s inter-temporal utility, Appendix B provides a detailed analysis of the information structures used in the paper, and Appendix C contains the proofs.

2 The principal-agent model

A risk-neutral principal owns a production technology that requires productive effort from an agent in \( N \) periods \( t = 1, \ldots, N \). The agent is risk- and effort-averse with exponential utility and quadratic effort cost of the form

\[
u(w, a) = -\exp[-r(w - \frac{1}{2}(a_1^2 + \cdots + a_N^2))],\]

where \( w \) is the agent’s terminal wealth and \( a = (a_1, \ldots, a_N) \) is the agent’s effort at the start of periods 1 through \( N \).\(^2\) The agent’s certainty equivalent of terminal wealth \( \bar{w} \) and effort \( a \) is, assuming \( \bar{w} \) to be normally distributed,

\[
ACE(\bar{w}, a_1, \ldots, a_N) = E[\bar{w}] - \frac{1}{2} r \text{var}(\bar{w}) - \frac{1}{2}(a_1^2 + \cdots + a_N^2).
\]

The output from agent’s effort \( a_t \in \mathbb{R} \) is, for \( t = 1, \ldots, N \), \( \bar{z}_t = b_t a_t + \bar{\lambda}_t \), where \( \lambda_t \) is an arbitrary mean zero noise term which does not depend on \( a_t \). The outcomes \( z_t \) are not observed until after the termination of the contract at the end of period \( N \). Hence, the output \( \bar{z}_t \) only determines the principal’s expected surplus, and since the principal is risk-neutral, no further distributional assumptions are needed regarding \( \bar{\lambda}_t \). The agent’s actions are unobservable. Hence, neither the output nor the agent’s actions are contractible.

A contractible performance measure \( x_t \) is observed at the end of each period. The agent’s effort in period \( t \) affects only the mean of the performance measure in that period, \( \bar{x}_t = m_t a_t + \bar{\varepsilon}_t \), where \( \varepsilon_t \) are mean zero noise terms. The noise terms in the performance measures are joint normally distributed with variance-covariance matrix \( \Sigma_N \).

There is more than one agent that the principal can employ in each period. All agents are

\(^2\)While multiplicative exponential utility without discounting appears to be a very restrictive framework, the same key qualitative results are obtained with time additive utility and discounting, see Appendix A.
identical (the agents have the same ability and the same utility functions) and have alternative employment opportunities. Each agent’s reservation certainty equivalent is normalized to zero in each period. Both the principal and the agents are assumed to have discount rates of zero. Utility functions, discount rates, reservation wages, the nature of the production technology, and the information structure are common knowledge.

The agent is either compensated by a long-term contract $c(\tilde{x}_1, \ldots, \tilde{x}_N)$ to be settled at the end of period $N$ or by a series of short-term contracts $(c_1(\tilde{x}_1), \ldots, c_N(\tilde{x}_1, \ldots, \tilde{x}_N))$ to be settled at the end of each period. Contracts are always assumed to be linear, and the only contractible information is given by the $N$ performance measures. The resulting contracts are thus the optimal linear contracts in each case, although linear contracts are not optimal.

Since $\tilde{x}_1, \ldots, \tilde{x}_N$ are joint normally distributed, it follows that the conditional distribution of $\tilde{x}_{t+1}, \ldots, \tilde{x}_N$ given $\tilde{x}_1, \ldots, \tilde{x}_t$ is also normal. For each $1 \leq t \leq N$, let $\tilde{a}_t = (a_1, \ldots, a_t)$, and $\tilde{x}_t = (x_1, \ldots, x_t)$ denote the histories of actions and outcomes from the first $t$ variables in the sequence. I will use the notation $E_t[\cdot]$ for conditional expectation given history $\tilde{x}_t$ and $\text{cov}_t(\cdot, \cdot)$ for conditional covariance given history $\tilde{x}_t$. Throughout the paper, the notation $m_t \tilde{a}_t$ refers to multiplication component by component, that is $m_t \tilde{a}_t = (m_1 a_1, \ldots, m_t a_t)$.

Since the means of the performance measures given past outcomes are influenced by managerial effort, conditional expectations will depend on either observed or conjectured past managerial effort. Specifically, the manager knows his past actions $\tilde{a}_t$, so from his perspective, $E_t[\cdot] = E_t[\cdot|\tilde{x}_t, \tilde{a}_t]$. The principal, on the other hand, does not observe the manager’s actions, but has conjectures about the agent’s past actions $\tilde{a}_t = (\tilde{a}_1, \ldots, \tilde{a}_t)$. Thus, from the principal’s perspective, $\tilde{E}_t[\cdot] = E_t[\cdot|\tilde{x}_t, \tilde{a}_t]$. In addition, the conditional expectations at time $t$ for any future performance measures $E_t[\tilde{x}_{t+k}]$ implicitly assume conjectured future actions $\tilde{a}_{t+k}$ both from the principal’s and the agent’s perspective. The conditional variance of $\tilde{x}_t$ given history $\tilde{x}_{t-1}$ is denoted $\sigma_t^2 = \text{var}(\tilde{x}_t|\tilde{x}_{t-1})$.\footnote{The conditional variances do not depend on the agent’s actions and represent the common posterior beliefs of the principal and the agent about the variance of future performance measures given past observations of the performance measures. In addition, note that while the conditional expectations depend on past observed values of the performance measures, conditional variances do not depend on these values, but only on time and the observability of these performance measures.}

Given linear contracts and normally distributed performance measures, the agent’s wealth is
normally distributed as well, which implies that the agent’s certainty equivalent of wealth and effort at the start of period $t$ is given by

$$\text{ACE}(\tilde{w}, \tilde{a}_t, \ldots, \tilde{a}_N|\underline{\tilde{a}}_{t-1}, \underline{\tilde{a}}_{t-1}) = E_{t-1}[\tilde{w}] - \frac{1}{2}r \text{var}_{t-1}(\tilde{w}) - \frac{1}{2}(\tilde{a}_1^2 + \cdots + \tilde{a}_N^2),$$

where $\underline{\tilde{a}}_{t-1}$ is the history of actions already taken, and $\tilde{a}_t, \ldots, \tilde{a}_N$ are the actions that the agent expects to take in periods $t, \ldots, N$. Throughout the paper I consider the actions $\underline{\tilde{a}}_{t-1}$ a sunk cost at the start of period $t$, and therefore not directly included in the agent’s certainty equivalent (1). Past actions impact the agent’s welfare at the start of period $t$ only through their impact on the means of past performance measures in the conditional expectation $E_{t-1}[\tilde{w}]$.

The agent’s wealth $\tilde{w}$ represents the total compensation to be received by the agent, and the agent is indifferent as to the timing of consumption. Thus, the agent’s utility ensures that there are no inter-temporal consumption smoothing issues. In addition, the agent’s exponential utility eliminates wealth effects, in that compensation paid (earned) does not impact the agent’s risk preferences. The quadratic cost of effort, additive across tasks, means that the agent is not indifferent to the allocation of effort among tasks in the $N$ periods.\footnote{The agent’s utility implies that, over $N$ periods, his certainty equivalent is a function of $(\tilde{a}_1^2 + \cdots + \tilde{a}_N^2)$, and not of total effort $a_1 + \cdots + a_N$.}

3 Full commitment, long-term contract

In this section, I assume that the principal can commit at the start of the first period to an $N$-period contract $\tilde{c} = a_0 + \beta_1 \tilde{x}_1 + \cdots + \beta_N \tilde{x}_N$. The terms of this contract are not subject to renegotiation. Furthermore, if the agent accepts the contract at the start of the first period, he commits for $N$ periods, and cannot leave in a later period. These assumptions about the parties’ ability to commit make the model equivalent (within the LEN framework) to a $N$ task, $N$ correlated performance measures, as analyzed by Feltham and Xie [12].

The principal’s problem is to maximize, at the start of the first period, the expected total outcome net of the agent’s compensation, subject to the agent’s participation constraint and the
agent’s incentive compatibility constraint. Let $M$ be a diagonal $N \times N$ matrix with $m_1, \ldots, m_N$ on the main diagonal. Then, the performance measures can be written in vector form as $\tilde{x} = Ma' + \tilde{c}$, where $a'$ is a column vector with components $a_1, \ldots, a_N$. Let $b = (b_1, \ldots, b_N)'$ be the column vector with components $b_1, \ldots, b_N$ such that the principal’s gross payoff is $\tilde{z} = b'a + \tilde{\lambda}_1 + \cdots + \tilde{\lambda}_N$. With this notation, the $N$-period full commitment contract is characterized by the following Proposition (see Feltham and Xie [12]).

**Proposition 1** The optimal linear contract $\tilde{c} = \alpha_0 + \beta_1 \tilde{x}_1 + \cdots + \beta_N \tilde{x}_N$ with full commitment for $N$ periods and the optimal actions are characterized by $a_1 = \beta_1 m_1, \ldots, a_N = \beta_N m_N$,

$$\beta = QMb ,$$

$$a = M\beta = MQMb ,$$

where $Q = (M^2 + r\Sigma_N)^{-1}$. The principal’s expected surplus is

$$U^p = b \cdot a - \frac{1}{2} r \text{var}(\beta_1 \tilde{x}_1 + \cdots + \beta_N \tilde{x}_N) - \frac{1}{2}(a_1^2 + \cdots + a_N^2) .$$

With full commitment to a long-term contract, the only role played by the correlation between adjacent performance measures is through the impact on the total risk to which the agent is exposed for incentive purposes as measured by $\text{var}(\tilde{c}) = \text{var}(\beta_1 \tilde{x}_1 + \cdots + \beta_N \tilde{x}_N)$. 

**4 Commitment and renegotiation**

In this section, I assume that the principal and the agent commit at the start of the first period to an $N$-period contract subject to renegotiation in subsequent periods. Specifically, the principal commits to an $N$-period contract and the agent commits to stay for $N$ periods if he finds the initial contract acceptable. However, the principal and the agent cannot commit not to renegotiate after the performance measures are observed. The terms of the contract are subject to renegotiation in the usual sense: the existing contract can only be replaced by a new contract if both parties agree
to it. The renegotiation takes the form of a take-it-or-leave-it offer by the principal.\footnote{The renegotiation concept I use here is the same as that of Fudenberg and Tirole [14] in that both parties must agree to the renegotiated contract, but the timing is different. In my model, renegotiation takes place after the performance measure \(x_t\) is observed, while Fudenberg and Tirole have the renegotiation take place between the time the agent takes the action and the time the performance measure is observed in a single period model. Having the renegotiation take place after \(x_t\) is observed and before \(a_{t+1}\) is taken avoids the insurance/adverse selection problem of Fudenberg and Tirole.}

The time line of events is as follows.

1) At the start of the first period, the principal offers the agent a linear contract based on the \(N\) contractible performance measures: 
\[
\tilde{c}^{I1} = \alpha_0^{I1} + \beta_1^{I1} \tilde{x}_1 + \cdots + \beta_N^{I1} \tilde{x}_N.
\]

2) If the agent accepts the contract, he commits to the terms of the contract, unless both parties agree later to replace it by a new contract.

3) At the start of period \(t\), the principal can make a renegotiation offer 
\[
\tilde{c}^{Rt} = \alpha_0^{Rt}(\tilde{x}_{t-1}) + \beta_t^{Rt}(\tilde{x}_{t-1}) \tilde{x}_t + \cdots + \beta_N^{Rt}(\tilde{x}_{T-1}) \tilde{x}_N.
\]
At this time, the contract under renegotiation is \(\tilde{c}^{I1}\), which is the outcome of period \(t-1\) renegotiation. The contract that is renegotiated consists of a fixed payment determined by the history of past performance measures and variable payments for the remaining performance measures that are still uncertain at the time of renegotiation. The renegotiation offer is then restricted to a contract of the same form, that is the principal may offer a different fixed payment and different variable payments only for the performance measures that are reported in the future. All payments, fixed and variable, may in general depend on past history.

4) After \(x_N\) has been reported, the contract in effect at the time is settled at the end of period \(N\).
that is linear in \( x_{t-1} \). In other words, the renegotiation offers are also linear in the \( N \) performance measures from an ex-ante (start of the first period) perspective.

An equilibrium in the principal-agent renegotiation game consists of a sequence of contracts \((\tilde{c}^{I1}, \tilde{c}^{R1}, \ldots, \tilde{c}^{IN}, \tilde{c}^{RN})\), the agent’s actions \( a_1, \ldots, a_N \), and principal’s beliefs about the agent’s actions \( \hat{a}_1, \ldots, \hat{a}_N \) such that: (i) the agent accepts both the initial contract and all the renegotiation offers, and rationally anticipates all renegotiation offers and their acceptance when selecting his actions; (ii) the principal’s beliefs are correct \( \hat{a}_t = a_t \) for all \( t \); (iii) the sequence \((\tilde{c}^{I1}, \tilde{c}^{R1}, \ldots, \tilde{c}^{IN}, \tilde{c}^{RN})\) is ex-ante (start of the first period) optimal and \((\tilde{c}^{I1}, \tilde{c}^{Rt}, \ldots, \tilde{c}^{IN}, \tilde{c}^{RN})\), is ex-post (start of period \( t \), for all \( t \), conditional on \( x_{t-1}, \hat{a}_{t-1} \)) optimal from the principal’s point of view.

The following proposition shows that the analysis of the equilibrium can be restricted without loss of generality to renegotiation-proof contracts.\(^6\)

**Proposition 2** If the sequence \((\tilde{c}^{I1}, \tilde{c}^{R1}, \ldots, \tilde{c}^{IN}, \tilde{c}^{RN}, a_1, \ldots, a_N)\) of contracts and actions is an equilibrium in the principal-agent renegotiation game and \( \tilde{c}^{Rt} \) is linear in \( x_{t-1}, \ldots, x_N \) at the time of renegotiation, then \( \tilde{c}^{Rt} \) is also ex ante (at the start of period \( 1, \ldots, t-1 \)) linear in \( \hat{x}_{t-1}, \ldots, \hat{x}_N \) and offering the contract \( \tilde{c}^{RN} \) in every period \((\tilde{c}^{RN}, \ldots, \tilde{c}^{RN}, a_1, \ldots, a_N)\) is an equivalent equilibrium with a single renegotiation-proof contract.

The main idea in the proposition is that if the contract is renegotiated in equilibrium, the agent’s actions are completely determined by the renegotiated contract. The principal cannot gain by offering the agent a contract that will later be renegotiated as long as the agent anticipates the renegotiation.

The principal’s problem is to maximize at the start of the first period the expected total outcome net of the agent’s compensation, subject to the agent’s participation constraint, and the agent’s incentive compatibility constraints. The contract must also be renegotiation-proof at the start of subsequent periods. The requirement that the contract is renegotiation-proof means that at the start of periods \( t = 2, \ldots, N \), the contract maximizes the principal’s expected total outcome net of the agent’s compensation conditional on information from periods \( 1, \ldots, t-1 \) and subject to the

\(^6\)The proposition does not follow from Fudenberg and Tirole [14] because of the linear contracts restriction. The proof is, however, similar.
agent’s participation constraint and the agent’s period \( t, \ldots, N \) incentive compatibility constraints.

**Proposition 3** Given the optimal linear \( N \)-period renegotiation-proof contract \( \tilde{c} = \alpha_0 + \beta_1 \tilde{x}_1 + \cdots + \beta_N \tilde{x}_N \), the optimal actions are characterized by \( a_t = m_t \beta_t \), for all \( t \) and

\[
\alpha_0 = -\frac{1}{2}(\alpha_1^2 + \cdots + \alpha_N^2) + \frac{1}{2} r \text{var}(\beta_1 \tilde{x}_1 + \cdots + \beta_N \tilde{x}_N). \tag{5}
\]

The principal’s surplus is given by

\[
U^p = b_1 a_1 + \cdots + b_N a_N - \frac{1}{2} r \text{var}(\beta_1 \tilde{x}_1 + \cdots + \beta_N \tilde{x}_N) - \frac{1}{2}(\alpha_1^2 + \cdots + \alpha_N^2). \tag{6}
\]

Explicit recursive formulas for the optimal actions will be derived in the next section, where a mechanism for replicating the renegotiation-proof solution by short-term contracts is presented.

The driving force behind renegotiation is the principal’s desire to optimally adjust the agent’s induced effort in subsequent periods to match the (reduced) ex-post variance of the performance measures. In this context, the fact that \( x_t \) is observed is important, and not the actual value of \( x_t \). As a result, period \( t, \ldots, N \) actions and period \( t, \ldots, N \) incentives are independent of \( x_t \), the agent’s actions \( a_t \), and the principal’s conjectures of the agent’s actions \( \tilde{a}_{t-1} \).

The inability of the principal to commit not to renegotiate is ex-ante inefficient relative to the full commitment case. The reason is that the renegotiation-proof contract imposes additional binding constraints on the optimal contract. In both cases, the principal chooses the agent’s actions to be induced by the contract in order to maximize his surplus as given by (6). In the full commitment case, the principal maximizes \( U^p \) unconstrained, while with renegotiation in periods \( 2, \ldots, N \), the contracts are constrained. If the performance measures in the \( N \) periods are independent, there is no difference between the two contracts, which take the form of repeatedly inducing the optimal action from the one period problem.
5 Fair Contracts

In this section, I generalize to \( N \) periods and an arbitrary information structure the solution to the agency problem with short-term contracts and commitment to fairness as described by Christensen, Feltham, and Şabac [4]. This idea of fair contracts is based on Baron and Besanko [1], see also Fehr, Gächter, and Kirchsteiger [11] and Rabin [23]. A contracting relationship is governed by fairness if the principal is restricted to fair wages and the agent must participate in all periods if he accepts the contract in the first period. Fair wages are paid when the agent gets his reservation wage as if he could leave in each period. That is, the agent’s certainty equivalent of future compensation, conditional on available information and conjectured actions is set to zero at the start of each period. Thus, in addition to the usual contract acceptance constraint at the start of the first period, there are additional constraints that the period \( t \) contract is acceptable to the agent as if the agent had other employment opportunities, had not committed to stay for all periods, and had taken the conjectured actions in periods \( 1, \ldots, t - 1 \). The key fact here is the agent’s ability to commit to stay for \( N \) periods; removing the agent’s ability to commit for all \( N \) periods leads to a situation where there is no equilibrium in which the agent stays in all \( N \) periods (see also Christensen, Feltham and Şabac [4] and Şabac [24]). The agent gives up his ability to leave in subsequent periods for the guarantee of fair compensation in periods \( 2, \ldots, N \).

The timeline of events is as follows.

1) At the start of the first period, the principal offers the agent a linear contract based on the first-period performance measure \( x_1 \): 
\[
\tilde{c}_1 = \alpha_1 + \beta_1 \tilde{x}_1.
\]

2) If the agent accepts the contract, he commits to stay for all \( N \) periods. The principal commits to offer the reservation certainty equivalent of wages in each period, given all the information available at the time of contracting and given that the agent’s actions correspond to the principal’s conjectures.

3) In the first period, after the agent has provided effort \( a_1 \) unobservable by the principal, the performance measure \( x_1 \) is publicly reported.

4) After the performance measure \( x_1 \) is publicly reported, the first-period contract is settled at the
end of the first period.

5) At the start of each period $t, t \geq 2$, a new contract is offered by the principal: $\tilde{c}_t = \alpha_t(x_{t-1}, \hat{a}_{t-1}) + \beta_t(x_{t-1}, \hat{a}_{t-1})\tilde{x}_t$. This contract is subject to the agent reservation wage restriction and is accepted by the agent (since I assumed commitment for all $N$ periods on the agent’s part). The terms of the period $t$ contract may depend on the past observed values of the performance measures and on the principal’s conjectures about the agent’s past actions $(x_{t-1}, \hat{a}_{t-1})$.

6) In period $t$, the agent provides effort $a_t$, then the performance measure $x_t$ is reported. The period $t$ contract is settled at the end of period $t$.

Short-term contracting with fair wages relies on the principal’s conjectures regarding the (unobservable) agent actions. At the start of period $t$, when $\tilde{c}_t$ is set, the terms of the contract depend on $E_{t-1}[\tilde{x}_t|x_{t-1}, \hat{a}_{t-1}]$, where $\hat{a}_{t-1}$ are the principal’s conjectures of the agent’s actions in periods $1, \ldots, t-1$. The concept of fair wages in period $t$ assumes that the principal’s conjectures are correct, that is the agent actually has provided effort $\hat{a}_{t-1}$ in the first $t-1$ periods. At the start of the first period, and any subsequent period $t$, fair wages refer to the total future compensation paid to the agent over the remaining periods $\tilde{c}_t + \cdots + \tilde{c}_N$, and involve both the principal’s and the agent’s conjectures of future actions $\hat{a}_t, \ldots, \hat{a}_N$. The principal and the agent are assumed to be in agreement over the conjectured future actions. For example, the principal states his conjecture $\hat{a}_1$ as part of the first-period contract $\tilde{c}_1$. The agent then agrees to that conjecture when accepting the initial contract, and that will be the basis for setting the second-period contract.\footnote{An alternative would be that both parties solve for the unique equilibrium conjecture $\hat{a}_1$. In this case, the principal can use an expert witness (professor of accounting) to prove that the second-period wage is fair.} While the agreement on conjectured actions restricts the principal’s contract choices (given fairness and past performance measures), the agent’s actions are unconstrained, even though, in equilibrium, the agent will choose to implement the conjectured actions.

Thus, an optimal sequence of contracts under commitment to fairness is also characterized by a rational expectations equilibrium regarding the agent’s actions. The agent’s commitment to stay for all $N$ periods is essential in sustaining this rational expectations equilibrium, as shown later in this section.
Let $\tilde{w}_t$ denote the agent’s cumulative compensation net of personal effort cost from period $t$ to period $N$

$$\tilde{w}_t = \tilde{c}_t - \frac{1}{2} \tilde{a}_t^2 + \cdots + \tilde{c}_N - \frac{1}{2} \tilde{a}_N^2.$$  \hspace{1cm} (7)

At the start of period $t$ (at contracting time), the manager’s objective is the certainty equivalent of $\tilde{w}_t$ conditional on all available information $x_{t-1}$, $\tilde{a}_{t-1}$, and on conjectures about future actions $\hat{a}_t, \ldots, \hat{a}_N$, denoted $\text{ACE}_{t-1}(\tilde{w}_t, \hat{a}_t, \ldots, \hat{a}_N|x_{t-1}, \tilde{a}_{t-1})$.

$$\text{ACE}_{t-1}(w_t, \hat{a}_t, \ldots, \hat{a}_N|x_{t-1}, \tilde{a}_{t-1}) = \mathbb{E}_{t-1}[\tilde{c}_t - \frac{1}{2} \tilde{a}_t^2] + \cdots + \mathbb{E}_{t-1}[\tilde{c}_N - \frac{1}{2} \tilde{a}_N^2] - \frac{1}{2} \mathbb{V}_{t-1}(\tilde{c}_t + \cdots + \tilde{c}_N).$$ \hspace{1cm} (8)

The values $\hat{a}_t, \ldots, \hat{a}_N$ represent at this time the agent’s conjectures of his future actions. The agent correctly anticipates that the terms of future contracts will depend on all history available at contracting time.

At the start of period $t$, the principal chooses $\alpha_t$ and $\beta_t$, taking into account all available information $x_{t-1}$ and his conjectures regarding the manager’s actions, past and future, $\hat{a}_1, \ldots, \hat{a}_N$. In a rational expectations equilibrium, at each point in time, the principal’s conjectures about past managerial actions are correct, and the principal and the manager have the same conjectures regarding the manager’s future actions, that is $a_t = \hat{a}_t$ for all $t$.

The following lemma shows that, conditional on time $t$ information, future performance measures adjusted for time $t$ conditional expectations are mutually independent.

**Lemma 1** For any $1 \leq t \leq k$, the following relations hold:

1. $\mathbb{E}_{t-1}[x_k - \mathbb{E}_{k-1}[x_k]] = 0$

2. $\mathbb{V}_{t-1}(x_k - \mathbb{E}_{k-1}[x_k]) = \mathbb{V}_{k-1}(x_k - \mathbb{E}_{k-1}[x_k])$

3. $x_t, x_{t+1} - \mathbb{E}_t[x_{t+1}], x_{t+2} - \mathbb{E}_{t+1}[x_{t+2}], \ldots$ are mutually independent conditional on $x_{t-1}$.

The next lemma shows that, in a rational expectations equilibrium, the fairness constraints $\text{ACE}_{t-1}(w_t) = 0$ imply $\text{ACE}_{t-1}(c_k) = 0$, that is fair wages can be set for each period separately.
Lemma 2 In equilibrium, if the fairness constraint is satisfied at the start of each period, that is if $ACE_{t-1}(w_t, \hat{a}_t, \ldots, \hat{a}_N|\xi_{t-1}, \hat{\xi}_{t-1}) = 0$ for all $1 \leq t \leq N$, then for any $t$ the following hold.

1. The agent’s certainty equivalent of future compensation at the start of period $t$ is the same as the certainty equivalent of period $t$ compensation

$$ACE_{t-1}(w_t, \hat{a}_t, \ldots, \hat{a}_N|\xi_{t-1}, \hat{\xi}_{t-1}) = E_{t-1}[c_t|\xi_{t-1}, \hat{\xi}_{t-1}, \hat{a}_t] - \frac{1}{2} \sigma_t^2 - \frac{1}{2} r \text{var}_{t-1}(c_t). \quad (9)$$

2. The fixed wage set by the principal is

$$\alpha_t = -\beta_t \hat{E}_{t-1}[x_t] + \frac{1}{2} \sigma_t^2 + \frac{1}{2} r \text{var}_{t-1}(c_t). \quad (10)$$

3. At the start of period $t$, the (rationally anticipated) contracts $c_t, \ldots, c_N$ are mutually independent.

Thus, fair contracts are mutually independent and fixed wages in period $t$ compensate the agent only for period $t$ incentive risk. The following proposition characterizes the optimal fair contracts.

Proposition 4 The actions induced by the sequence of optimal fair contracts and the sequence of optimal explicit incentives are given by the following recursive relations:

$$a_N = m_N \beta_N = \frac{m_N^2 b_N}{m_N^2 + r \sigma_N^2}, \quad (11)$$

$$a_t = b_t - \frac{r \beta_t \sigma_t^2}{m_t}, \quad (12)$$

$$\beta_t = \frac{m_t b_t}{m_t^2 + r \sigma_t^2} + \frac{m_t^2 \sum_{k=t+1}^N \beta_k R_{k-1}^t}{m_t^2 + r \sigma_t^2}, \quad (13)$$

where $R_{k-1}$ characterizes the conditional expectations operator, $E_{k-1}[\xi_k] = R_{k-1} \cdot \xi_{k-1}$, and $R_{k-1} = (R_{k-1}^1, \ldots, R_{k-1}^k, \ldots, R_{k-1}^{k-1})$. The fixed part of the agent’s compensation is

$$\alpha_t = -\beta_t \hat{E}_{t-1}[x_t] + \frac{1}{2} \sigma_t^2 + \frac{1}{2} r \sigma_t^2 \beta_t^2. \quad (14)$$
As in the two-period case, the optimal short-term contracts under commitment to fairness are equivalent to the optimal renegotiation-proof contract (see also Gibbons and Murphy [15] and Christensen, Feltham, and Şabac [4]).

**Proposition 5** The equilibria from Proposition 3 and Proposition 4 are equivalent in that the agent takes the same actions, and the principal and the agent have the same payoffs.

If the agent cannot commit to stay for all $N$ periods, then there is no equilibrium with a sequence of short-term contracts in which the agent stays for all $N$ periods. As a result, the commitment to fairness assumption is not only sufficient, but also necessary to obtain the solution described in Proposition 4. The fairness assumption is necessary since, once the agent has committed to stay for all $N$ periods, there must be a restriction in subsequent periods to prevent an “infinite” transfer from the agent to the principal through the period $t \geq 2$ contracts. The anticipation of such a contract renders any first-period contract in which the agent commits for all $N$ periods unacceptable to the agent. Thus, the simplest solution to this problem is a lower bound on the agent’s reservation certainty equivalent, which is precisely what the fairness constraint provides. A detailed analysis of commitment assumptions and the implementation of the renegotiation-proof equilibrium with short-term contracts for two periods can be found in Christensen, Feltham, and Şabac [4]. Their results can be extended to $N$ periods in a straightforward manner.

6 Dynamic incentives

In this section, I examine the impact of the information structure – as represented by the covariance matrix of the performance measures $\Sigma_N$ – on incentive dynamics and induced managerial effort. The induced managerial effort is the same for both fair contracts and the renegotiation-proof contract, while the explicit incentives differ. With a renegotiation-proof contract there are no implicit incentives, while with fair contracts implicit incentives for each period are included in future periods’ fixed wages. For an extensive discussion of implicit incentives and contracting form in the two-period case of the model considered in this paper, see Christensen, Feltham and Şabac [4]. The discussion below generalizes the results on implicit incentives of Gibbons and Murphy [15] to
a general information structure with the notable difference that I allocate all bargaining power to
the principal.

To provide an explicit analysis, I consider two particular correlation structures: one that corre-
sponds to the career concerns model of Gibbons and Murphy [15], and one that limits inter-period
correlation to adjacent periods.

The first information system, the career concerns model, denoted \( \eta^1 \), is characterized by

\[
\tilde{\varepsilon}_t = \tilde{\delta}_t + \tilde{\theta}, \tag{15}
\]

where the terms \( \tilde{\delta}_t \) are independent identically distributed with mean zero and variance \( \sigma_{\tilde{\delta}}^2 \), while
\( \tilde{\theta} \) is independent of the \( \tilde{\delta}_t \), has mean zero and variance \( \sigma_{\tilde{\theta}}^2 \). Thus, \( \tilde{\delta}_t \) represents period-specific
noise that is independent of all other noise components, while \( \tilde{\theta} \) represents managerial ability or
the match between manager and job, and is the same in all periods. The manager’s ability or job
match is not known to either the principal or the manager prior to contracting. The presence of
\( \tilde{\theta} \) implies that any two performance measures are positively correlated since \( \text{cov}(\tilde{x}_k, \tilde{x}_l) = \sigma_{\tilde{\theta}}^2 \) for
\( k \neq l \).

The second information system, the accounting model, denoted \( \eta^2 \), is characterized by

\[
\begin{align*}
\tilde{\varepsilon}_{t-1} &= \tilde{\delta}_{t-1} + \rho \tilde{\theta}_{t-2} + \tilde{\theta}_{t-1} \tag{16} \\
\tilde{\varepsilon}_t &= \tilde{\delta}_t + \rho \tilde{\theta}_{t-1} + \tilde{\theta}_t \tag{17} \\
\tilde{\varepsilon}_{t+1} &= \tilde{\delta}_{t+1} + \rho \tilde{\theta}_t + \tilde{\theta}_{t+1} \tag{18},
\end{align*}
\]

where \( \rho = \pm 1 \), \( \tilde{\delta}_t \), are period-specific, and \( \tilde{\theta}_t \) is a common component in adjacent periods. The \( \tilde{\delta}_t \),
\( \tilde{\theta}_t \) terms are mutually independent, have mean zero, \( \text{var}(\tilde{\delta}_t) = \sigma_{\delta}^2 \), and \( \text{var}(\tilde{\theta}_t) = \sigma_{\theta}^2 \). With positive
correlation, \( \rho = 1 \), and the common (between adjacent periods) component \( \tilde{\theta}_t \) can be thought of
as a shock that persists from one period to the next period. The noise component \( \tilde{\theta}_t \) first appears
in period \( t \), persists for one period and then disappears. In period \( t + 1 \), a new noise term \( \tilde{\theta}_{t+1} \)
appears, and so on.
With negative correlation, $\rho = -1$, and the common (between adjacent periods) component $\tilde{\theta}_t$ can be thought of as period $t$ accrual estimation errors that have to be reversed in the next period, $t + 1$. Thus, negatively correlated noise in accounting-based performance measures reflects the nature of the accrual process. In this model, however, the accrual estimation errors, as with all the other components of the noise in the performance measure, are outside the manager’s control.\footnote{Christensen, Demski, and Frimor \cite{2} and Demski and Frimor \cite{7, 8} examine the role played by income smoothing, or earnings management in dynamic incentives with renegotiation.}

The two information systems coincide in the two periods case for positive correlation. In the two periods case, Christensen, Feltham, and Şabac \cite{5} show that the principal’s surplus decreases in the correlation of the performance measures, and that accrual estimation errors may counteract the negative effects of having a high variance of managerial ability or job matching. Their findings suggest the correlation of performance measures may be endogenously determined. By contrast, in this paper, I assume the correlation structure to be exogenously given.

The following proposition gives explicit recursive equations for optimal actions and incentives derived from Proposition 4 using the two information systems described above.

**Proposition 6** For both information systems, the last period action and explicit incentive are determined by

$$a_N = m_N \beta_N = \frac{m_N^2 b_N}{m_N^2 + r \sigma_N^2}.$$ (19)

For information system $\eta^1$, the actions induced by the sequence of optimal fair contracts and the sequence of optimal explicit incentives are given by the following recursive relations:

$$\beta_t = \frac{m_t b_t - m_t^2 m_{t+1} b_{t+1}}{m_t^2 + r \sigma_t^2} + \frac{m_t^2}{m_t^2 + r \sigma_{t+1}^2} \frac{\sigma_t^2 m_{t+1}^2 + r \sigma_{t+1}^2}{m_t^2 + r \sigma_t^2} \beta_{t+1}$$ (20)

$$a_t = \frac{m_t^2 b_t - \sigma_t^2 m_t m_{t+1} b_{t+1}}{m_t^2 + r \sigma_t^2} + \frac{\sigma_t^2}{\sigma_{t+1}^2 + \sigma_t^2} \frac{m_t m_{t+1} + r \sigma_t^2}{m_t^2 + r \sigma_t^2} a_{t+1}$$ (21)

For information system $\eta^2$, the actions induced by the sequence of optimal fair contracts and the
sequence of optimal explicit incentives are given by the following recursive relations:

\[ \beta_t = \frac{m_t b_t + \rho \sigma_t^2 \frac{m_t^2}{m_{t+1}^2} m_{t+1} b_{t+1} + \frac{\sigma_t^2}{\sigma_{t+1}^2} \frac{m_t^2}{m_{t+1}^2} \beta_{t+1}}{m_t^2 + r \sigma_t^2} - \rho \sigma_t^2 r \frac{m_t^2}{m_t^2 + r \sigma_t^2} \beta_{t+1} \]  
(22)

\[ a_t = \frac{m_t^2 b_t}{m_t^2 + r \sigma_t^2} - \rho \sigma_t^2 r \frac{m_{t+1}^2}{m_t^2 + r \sigma_t^2} a_{t+1} \cdot \]  
(23)

In both cases, the fixed part of the agent’s compensation is

\[ a_t = -\beta_t E_{t-1} [\tilde{\varepsilon}_t] + \frac{1}{2} \sigma_t^2 + \frac{1}{2} r \sigma_t^2 \beta_t^2 \cdot \]  
(24)

To capture the idea of repeated agency and to simplify the analysis, I assume that all periods are identical, \( m_t = m, b_t = b \). With this simplification, the last period action and explicit incentive are

\[ a_N = m \beta_N = \frac{m^2 b}{m^2 + r \sigma_N^2} \]  
(25)

For information system \( \eta^1 \), equations (20)–(21) imply

\[ \beta_t = \frac{m^2 \sigma_t^2 + r \sigma_{t+1}^2}{m^2 + r \sigma_t^2} \beta_{t+1} \]  
(26)

\[ a_t = \frac{m^2 b \left( 1 - \frac{\sigma_t^2}{\sigma_{t+1}^2} \frac{\sigma_t^2}{\sigma_{t+1}^2} \right)}{m^2 + r \sigma_t^2} + \frac{\sigma_t^2 \sigma_{t+1}^2}{m^2 + r \sigma_t^2} a_{t+1} \cdot \]  
(27)

It is useful to consider the impact of the agent’s remaining tenure with the firm separately from the impact of decreasing posterior variances of the performance measures. To that end, assume that \( \sigma_t = \sigma_\infty \), that is consider periods that are far enough from the first so that the posterior variance is close to its limit value. In this case, the principal’s and the agent’s problem are approximately equivalent to the case of uncorrelated periods with \( \tilde{\varepsilon}_t = \tilde{\delta}_t \). Thus, towards the end of the agent’s tenure incentives and effort are close to the incentive and effort level from the single period problem with \( \tilde{\varepsilon} = \tilde{\delta} \). The negative effect of the agent’s human capital insurance drives the agent’s optimal
effort level significantly below last periods’ effort at the start of his tenure.

It is straightforward to show, from equation (26) and the explicit expressions for the posterior variance that explicit incentives are decreasing. As a result, using equation (12), it follows that the effective incentives are increasing. While optimal agent effort and effective incentives increase over time as described by Gibbons and Murphy [15], the fact that explicit incentives decrease over time is due to the fact that I assume the principal has all bargaining power. Gibbons and Murphy’s opposite conclusions on explicit incentives are due to the fact that, in their model, the agent has all the bargaining power. For empirical research, it is noteworthy that the same optimal effort is induced by contracts whose explicit incentives are very different.

Numerical examples are provided in Figure 1. In all the graphs, I have assumed \( b_t = m_t = r = 1 \) and \( N = 15 \). In the career concerns model, I have \( \sigma^2_\delta = \sigma^2_\theta = 1/2 \). In the accounting model, I have \( \sigma^2_\delta = 0 \) and \( \sigma^2_\theta = 1/2 \). In both cases, these assumptions imply that \( \text{var}(\varepsilon_t) = 1 \) and the correlation between periods is \( 1/2 \) in the career concerns model and \( \pm 1/2 \) in the accounting model, respectively. Optimal actions and effective/explicit incentives in the career concerns model are presented in Figure 1(e)–(f).

For information system \( \eta^2 \), equations (22)–(23) imply

\[
\beta_t = mb \frac{1 + \rho \sigma^2_\eta}{\sigma^2_t} - \rho \sigma^2_\eta \frac{\sigma^2_{t+1}}{\sigma^2_t} \beta_{t+1} \tag{28}
\]

\[
a_t = \frac{m^2 b}{m^2 + r \sigma^2_t} - \frac{\rho \sigma^2_\eta}{m^2 + r \sigma^2_t} a_{t+1}. \tag{29}
\]

For the last periods of the agent’s tenure, the posterior variance is approximately equal to its limit value and equations (28) and (29) imply

\[
a_t - a_{t-1} = -\frac{\rho \sigma^2_\eta}{m^2 + r \sigma_\infty}(a_{t+1} - a_t) \tag{30}
\]

\[
\beta_t - \beta_{t-1} = -\frac{\rho \sigma^2_\eta}{m^2 + r \sigma_\infty}(\beta_{t+1} - \beta_t). \tag{31}
\]

Thus, the incentive insurance effect drives the agent’s incentives in the last periods as follows.
Figure 1: Optimal actions and explicit/effective incentives, multiplicative agent utility: $b_t = m_t = r = 1$ and $N = 15$ in all cases.
For positive correlation, $\rho = 1$, both sequences are alternating. This is due to the fact that, to reduce aggregate risk imposed on the agent the principal must choose a lower or a higher incentive depending on how high or low the next period’s incentive will be. For negative correlation, $\rho = -1$, the principal can choose higher incentives in previous periods due to the increasing beneficial effect of mutual insurance offered by future performance measures.

Moreover, moving backwards in time and holding the conditional variance constant, the agent’s effort and incentives converge quickly towards limit values. For positive correlation, these limit values are significantly below last period’s values. For negative correlation, the opposite holds, and the limit values are significantly above last period’s values. In both cases, explicit incentives are almost the mirror image of effective incentives because they are adjusted to take into account future implicit incentives. Finally, during the first periods of the agent’s tenure, the dominant effect is the reduction of posterior variances for the performance measures. Optimal actions and effective/explicit incentives in the accounting model are presented in Figure 1(a)–(d).

To conclude, the manager’s performance and incentives are driven by the reduction in posterior variance during the first few periods, are constant during the “middle” periods, and are driven by a last period insurance effect during the last few periods. Interestingly, for the negative correlation case, performance has an inverted U shape, and first-period performance is higher than last-period performance. For the positive correlation case, except for the last few periods, performance generally increases towards the last period and last-period performance is higher than first-period performance as in the career incentives model.

7 Managerial tenure

While in the preceding section I discussed the manager’s performance and incentives for a given tenure or contracting horizon, in this section I consider the principal’s preferences for different contracting horizons and the possibility of optimal tenure for the agent. Given that closed-form expressions of the principal’s surplus for a general agent tenure $N$ are not tractable, I will illustrate the main points with a few numerical examples.
Note that for $N = 2$, the career concerns model and the accounting model with positive correlation coincide. The principal’s preferences in the absence of switching costs are for turnover every period with positive correlation and maximum tenure for negative correlation (see Christensen, Feltham and Šabac [4] and Dutta and Reichelstein [10]). Of course, with only two periods there are only corner solutions: a model with more than two periods is necessary to examine the issue of non-degenerate optimal tenure.

From Proposition 4, it follows that

$$
\tilde{c}_t = \beta_t (\tilde{x}_t - E_{t-1}[\tilde{x}_t]) + \frac{1}{2} a_t^2 + \frac{1}{2} r \sigma_t \beta_t^2,
$$

which implies

$$
E_0[\tilde{c}_t - \tilde{c}_t] = b_t a_t - \frac{1}{2} a_t^2 - \frac{1}{2} r \sigma_t \beta_t^2,
$$

since $E_0 [\tilde{x}_t - E_{t-1} [\tilde{x}_t]] = 0$ for all $1 \leq t \leq N$. Using $a_t = b_t - r \sigma_t m_t^{-1} \beta_t$ to substitute for $\beta_t$ in equation (33) gives the principal’s ex-ante expected surplus for period $t$

$$
E_0[U^p_t] = b_t a_t - \frac{1}{2} a_t^2 - \frac{1}{2} r \sigma_t \frac{m_t^2}{r^2 \sigma_t^2} (b_t - a_t)^2.
$$

Since

$$
\frac{\partial E[U^p_t]}{\partial a_t} = \left( 1 + \frac{m_t^2}{r \sigma_t} \right) (b_t - a_t) \geq 0
$$

for all $a_t \leq b_t$, it follows that the expected period $t$ profit is strictly increasing in managerial effort and attains a maximum at $a_t = b_t$ (which is first-best in this case).

Since I need to compare different lengths of managerial tenure from the point of view of a long-lived firm, I assume that the firm is infinitely lived and decides on a stationary tenure policy, that is managers are replaced every $N$ periods in perpetuity. Each time a new agent is hired, prior history is assumed to be irrelevant.\(^9\) The principal’s objective function is the average per period total surplus $U^p/N = \sum_{t=1}^{N} U^p_t / N$, which corresponds to the annual equivalent value of a stationary

\(^9\)This may be either because all noise that is not period-specific is agent-specific, or because a long enough prior history is available which makes the conditional information structure identical at the time a new agent is hired, see below.
policy of hiring agents every $N$ periods in perpetuity.

In all the graphs in Figure 2, I have assumed $b_t = m_t = r = 1$. The manager’s tenure is $1 \leq N \leq 35$. In the career concerns model, I have $\sigma^2_\beta = \sigma^2_\theta = 1/2$. In the accounting model, I have $\sigma^2_\beta = 0$ and $\sigma^2_\theta = 1/2$. In both cases, these assumptions imply that $\text{var}(\varepsilon_t) = 1$ and the correlation between periods is $1/2$ in the career concerns model and $\pm 1/2$ in the accounting model, respectively.

Figure 2(a) and Figure 2(b) show that in the accounting model with negative correlation, the principal’s welfare increases with the agent’s tenure, while for positive correlation, the principal’s welfare decreases with the agent’s tenure, giving the same preferences for corner solutions as in the two-period case. Figure 2(c) is similar to 2(b) in that the principal’s welfare decreases in agent tenure. Thus, internal optimal tenure is possible only with positively correlated performance measures and Figures 2(d),(e),(f) provide values for the switching cost that give internal optima. By continuity, all results hold in a neighborhood of the given parameter values. With the exception of Figure 2(e), all results also hold in the case of time additive utility with small discount rates, see Appendix A.

The above results can be summarized as follows (renegotiation is assumed in all cases). First, absent a switching cost, or if the switching cost is small enough when hiring a new manager, there is no interior optimal managerial tenure, turnover every period is preferred with positively correlated performance measures. Second, there is a range of switching costs for which there is interior optimal tenure with positively correlated measures. Third, if the switching cost is large enough, maximum possible tenure is preferred with positively correlated performance measures. Finally, maximum possible tenure is preferred with negatively correlated performance measures regardless of switching costs.

The principal’s average surplus is influenced by the switching cost, a “retirement horizon”, and a “learning effect” in the first periods. While the switching cost and the learning effect make the principal prefer longer managerial tenure in all cases, the retirement horizon’s impact depends on the correlation of performance measures. For negatively correlated performance measures the cost of incentive risk increases towards retirement and with positively correlated performance measures
(a) Average principal’s surplus, accounting model, \( \sigma^2 = 0, \sigma^2 = 1/2, \rho = -1 \).

(b) Average principal’s surplus, accounting model, \( \sigma^2 = 0, \sigma^2 = 1/2, \rho = 1 \).

(c) Average principal’s surplus, career concerns, \( \sigma^2 = \sigma^2 = 1/2, c = 0 \).

(d) Average principal’s surplus, accounting model, renegotiation, \( \sigma^2 = 0, \sigma^2 = 1/2, \rho = 1, c = 0.13 \).

(e) Average principal’s surplus, career concerns, commitment, \( \sigma^2 = \sigma^2 = 1/2, c = 0.3 \).

(f) Average principal’s surplus, career concerns, renegotiation, \( \sigma^2 = \sigma^2 = 1/2, c = 0.7 \).

Figure 2: Principal’s average surplus and tenure, multiplicative agent utility, \( b_t = m_t = r = 1 \), and \( N = 1, \ldots, 35 \) in all models.
the cost of incentive risk decreases towards retirement. Thus, all else equal, the cost of incentive risk is lower for longer tenure with negative correlation, and higher with positive correlation.

The principal’s tradeoff with positive correlation is between the higher incentive risk cost given by a longer retirement horizon on one hand, and the ability to spread the switching cost over several periods together with the reduction in posterior variances of the performance measures on the other hand.

The main implication for empirical research is that managerial retention policies may be endogenous and related to the time-series properties of the performance measures used in contracting. This holds true even if managerial retention and information system characteristics are simultaneously chosen, as long as observed performance measures are optimally chosen by the principal.

8 Conclusions

This paper examines the role played by the agent’s horizon in a dynamic principal agent relationship. After providing explicit solutions for optimal contracts in an $N$-period LEN model with renegotiation, I derive the behaviour of implicit/effective incentives and managerial performance for the entire duration of the contractual relationship and highlight the significant differences between different information environments. Negatively correlated performance measures lead to an inverted U shape pattern of performance, increasing during the first periods, and declining in the last periods of tenure. Positively correlated performance measures lead to increasing performance throughout the agent’s tenure. These effects are due to the cost of incentive risk the principal must compensate the agent for and given that renegotiation limits the use of performance measures from different periods for insuring the agent against incentive risk.

Not surprisingly, the agent’s effort and the principal’s surplus are higher with negatively correlated performance measures, due to the incentive risk insurance effects. As a result, longer tenure is always preferred if performance measures are negatively correlated, while the reverse holds true for positive correlation. In the presence of a fixed switching cost, the preference for longer tenure is reinforced with negative correlation, while optimal tenure longer than one period may obtain
with positive correlation. Specifically, with positive correlation, depending on the magnitude of the switching cost, turnover every period is optimal, turnover every $1 < N < \infty$ periods is optimal, or longer tenure is always preferred.

The main implications for empirical research are that both managerial performance and turnover are endogenous and depend on performance measure characteristics. Thus, performance around CEO turnover should depend on the time-series properties of the performance measures used in incentive contracts. If accounting-based performance measures exhibit negative auto-correlation, then performance of CEO’s whose incentives are accounting-based will exhibit an inverted U shape, and tenure will be longer; declining performance prior to turnover followed by increased performance by the successor is expected. If career concerns prevail, that is performance measures exhibit strong positive auto-correlation ex ante due to uncertain managerial quality or job match, performance is increasing during the manager’s tenure and tenure will be shorter; performance is increasing prior to turnover and the successor’s performance is lower and increasing as well. The model cannot be applied to “forced turnovers” where the manager is fired for poor performance and is more descriptive of normal, or anticipated, turnover, such as retirement.

Appendix A: Multiple consumption dates model

In this appendix, I present the alternative results for a multiple consumption date model with time additive utility and infinite consumption horizon for the agent. For detailed descriptions of this model, see Dutta and Reichelstein [9] and Christensen et al. [3]. The agent’s utility for a consumption stream $(c_t, c_{t+1}, \ldots)$ is

$$u_t(c) = -\sum_{k=t}^{\infty} \gamma^{k-t} \exp(-\hat{r} c_k),$$

where $w_t$ represents the agent’s consumption at date $t$, the start of period $t + 1$ and $\gamma = (1 + R)^{-1}$ is the discount rate. The principal is risk neutral and has the same discount rate as the agent. The agent can borrow or lend at rate $R$ and incurs a personal effort cost $\kappa_t = \frac{1}{2} w_t^2$ in each period.
Let $W_t$ and $K_t$ represent the NPV of future compensation and effort cost, respectively for each employment date $t = 1, \ldots, N$:

$$W_t = \sum_{k=t}^{N} \gamma^{k-t} w_t \quad \text{and} \quad K_t = \sum_{k=t}^{N} \gamma^{k-t} k_t.$$ 

It follows that the agent’s expected utility at date $t - 1$ is characterized by the certainty equivalent, see Christensen et al. [3]:

$$CE_{t-1} = \gamma E_{t-1}[W_t - K_t] - \frac{1}{2} r \sum_{k=t}^{N} \gamma^{k-t+1} \text{var}_{k-1}(E_k[W_k]),$$

where $r = (1 - \gamma)\hat{r}$. Given the above assumptions, the modifications for Proposition 4 are relatively straightforward, while all other results, except the full commitment contract characterization, remain unchanged. Specifically, equation (13) is replaced by

$$\beta_t = \frac{m_t b_t}{m_t^2 + r\sigma_t^2} + \frac{m_t^2}{m_t^2 + r\sigma_t^2} \sum_{k=t+1}^{N} \gamma^{k-t} \beta_{k+1} R_{k-1}^t.$$ 

Consequently, the recursive relations (20) and (21) in Proposition 6 for information system $\eta^1$ become

$$\beta_t = \frac{m_t b_t - \gamma m_t^2}{m_{t+1}^2 + \sigma_{t+1}^2 m_{t+1} b_{t+1}} + \frac{m_t^2}{m_{t+1}^2 + \sigma_{t+1}^2 m_{t+1} b_{t+1}} \left( \frac{\sigma_t^2}{\sigma_{t+1}^2} \frac{m_{t+1}^2 + \sigma_t^2 m_{t+1} b_{t+1}}{m_{t+1}^2 + \sigma_t^2 m_{t+1} b_{t+1}} \right) \beta_{t+1}$$

$$a_t = \frac{m_t^2 b_t - \gamma \sigma_t^2 m_t m_{t+1} b_{t+1}}{m_t^2 + \sigma_t^2} + \gamma \frac{\sigma_t^2}{\sigma_{t+1}^2} \frac{m_t m_{t+1} + \sigma_t^2 m_{t+1}}{m_t m_{t+1} + \sigma_t^2 m_{t+1}} a_{t+1}.$$ 

For information system $\eta^2$, the recursive relations (22) and (23) become

$$\beta_t = \frac{m_t b_t + \gamma m_t^2}{m_{t+1}^2 + \sigma_t^2} \frac{m_t^2}{m_{t+1}^2 + \sigma_t^2 m_{t+1} b_{t+1}} - \gamma \frac{\sigma_t^2}{\sigma_{t+1}^2} \frac{m_t^2}{m_{t+1}^2 + \sigma_t^2 m_{t+1} b_{t+1}} \beta_{t+1}$$

$$a_t = \frac{m_t^2 b_t}{m_t^2 + r\sigma_t^2} - \gamma \frac{\rho \sigma_t^2}{m_t^2 + r\sigma_t^2} a_{t+1}.$$
In particular, setting \( \gamma = 1 \) in the above equations formally gives the old expressions for multiplicative utility without discounting.\(^{10}\) Thus, for small enough discount rates, the qualitative results obtained in the paper do not change. For comparison purposes, I present below in Figure 3 and Figure 4 the analogous results to those in Figure 1 and Figure 2.

In all the graphs, I have assumed \( b_t = m_t = r = 1 \). The discount rate is set at \( R = 5\% \) in all models. In the career concerns model, I have \( \sigma_0^2 = \sigma_1^2 = 1/2 \). In the accounting model, I have \( \sigma_0^2 = 0 \) and \( \sigma_1^2 = 1/2 \). In both cases, these assumptions imply that \( \text{var}(\varepsilon_t) = 1 \) and the correlation between periods is 1/2 in the career concerns model and \( \pm 1/2 \) in the accounting model.

**Appendix B: Information environment, special cases**

In this section, I characterize the posterior beliefs for the two special cases considered in the paper, the career concerns model \( \eta^1 \) and the accounting model \( \eta^2 \). Since the performance measures are joint normally distributed, the conditional expectations, given past observations of the performance measures and conjectured past actions are linear in the available history:

\[
E_k[x_k] = m_k a_k + R_{k-1} \cdot (x_{k-1} - m_{k-1} a_{k-1})
\]

where the vector \( R_{k-1} \) characterizes the conditional expectation operator.

I now turn to the determination of explicit formulas for the conditional expectation and variance for the two information systems, given a history of performance measures and conjectured managerial actions.

For information system \( \eta_1 \), let \( \zeta_k = (\varepsilon_k - E_{k-1}[\varepsilon_k])/\text{var}(\varepsilon_k - E_{k-1}[\varepsilon_k])^{1/2} \). The random variable \( \zeta_k \) is independent of \( \varepsilon_1, \ldots, \varepsilon_{k-1} \) and normalized to have variance one. The conditional expectation of \( \varepsilon_{k+1} \) can then be calculated as

\[
E_k[\varepsilon_{k+1}] = E_{k-1}[\varepsilon_{k+1}] + E[\varepsilon_{k+1} | \zeta_k] = E_{k-1}[\varepsilon_k] + \text{cov}(\varepsilon_{k+1}, \zeta_k) \zeta_k \tag{40}
\]

\(^{10}\)One problem in the comparison with infinite consumption horizons is the impact of discount rates on the agent’s effective risk aversion, and in particular the fact that the agent is effectively risk-neutral if \( \gamma = 1 \).
Figure 3: Optimal actions and explicit/effective incentives, time additive agent utility: $b_t = m_t = r = 1$, $R = 5\%$, and $N = 15$ in all cases.
Figure 4: Principal’s average surplus and tenure, time additive agent utility, $b_t = m_t = r = 1$, $R = 5\%$, and $N = 1, \ldots, 35$ in all models.
where I have used $E_{k-1}[\varepsilon_{k+1}] = E_{k-1}[\varepsilon_k]$ due to the fact that the correlation structure is the same in both cases.

I now show that, for $k \geq 2$,

$$E_{k-1}[\varepsilon_k] = \frac{\sigma_0^2}{(k-1)\sigma_0^2 + \sigma_\delta^2} (\varepsilon_1 + \cdots + \varepsilon_{k-1}). \quad (41)$$

For $k = 2$, equation (41) follows from

$$E_1[\varepsilon_2] = \text{cov}(\varepsilon_2, \zeta_1)\zeta_1 = \text{cov} \left( \frac{\varepsilon_2}{\sigma_1}, \frac{\varepsilon_1}{\sigma_1} \right) \frac{\varepsilon_1}{\sigma_1} = \frac{\sigma_0^2}{\sigma_0^2 + \sigma_\delta^2} \varepsilon_1.$$

Assuming (41) to hold for $k - 1$, I will prove that it holds for $k$. From the induction hypothesis (41) it follows that

$$\varepsilon_k - E_{k-1}[\varepsilon_k] = \theta + \delta_k - \frac{\sigma_0^2}{(k-1)\sigma_0^2 + \sigma_\delta^2} ((k-1)\theta + \delta_1 + \cdots + \delta_{k-1})$$

$$= \frac{\sigma_\delta^2 \theta - \sigma_0^2 (\delta_1 + \cdots + \delta_{k-1}) + [(k-1)\sigma_0^2 + \sigma_\delta^2] \delta_k}{(k-1)\sigma_0^2 + \sigma_\delta^2}.$$

Consequently, the variance is given by

$$\sigma_k^2 = \text{var}(\varepsilon_k - E_{k-1}[\varepsilon_k]) = \frac{k\sigma_0^2 + \sigma_\delta^2}{(k-1)\sigma_0^2 + \sigma_\delta^2}. \quad (42)$$

From (40) it follows that

$$E_k[\varepsilon_{k+1}] = E_{k-1}[\varepsilon_k] + \text{cov}(\varepsilon_{k+1}, \zeta_k) \zeta_k$$

$$= E_{k-1}[\varepsilon_k] + \frac{\sigma_0^2}{k\sigma_0^2 + \sigma_\delta^2} (\varepsilon_k - E_{k-1}[\varepsilon_k]) \quad (43)$$

$$= \frac{\sigma_0^2}{k\sigma_0^2 + \sigma_\delta^2} (\varepsilon_1 + \cdots + \varepsilon_k).$$

The conditional expectation operator is then determined by

$$R_{k-1} = \frac{\sigma_0^2}{(k-1)\sigma_0^2 + \sigma_\delta^2}. \quad (44)$$
and the conditional variance is
\[
\sigma_k^2 = \sigma_\theta^2 \frac{k \sigma_\theta^2 + \sigma_\alpha^2}{(k-1) \sigma_\theta^2 + \sigma_\alpha^2} = \sigma_\theta^2 + \frac{\sigma_\theta^2 \sigma_\alpha^2}{(k-1) \sigma_\theta^2 + \sigma_\alpha^2}.
\]

(45)

The conditional variances converge in the limit to \(\sigma_\alpha^2 = \sigma_\theta^2\) as the uncertainty over \(\theta\) is eliminated.

For information system \(\eta_2\), the period \(k + 1\) noise term \(\varepsilon_{k+1}\) is independent of the history \(\varepsilon_1, \ldots, \varepsilon_{k-1}\). It follows that \(E_k[\varepsilon_{k+1}] = E[\varepsilon_{k+1}|\varepsilon_k - E_{k-1}[\varepsilon_k]]\). Since \(\sigma_k^2 = \text{var}(\varepsilon_k - E_{k-1}[\varepsilon_k])\) and \(\text{cov}(\varepsilon_{k+1}, \varepsilon_k - E_{k-1}[\varepsilon_k]) = \text{cov}(\varepsilon_{k+1}, \varepsilon_k) = \rho \sigma_\theta^2\), it follows that
\[
E_k[\varepsilon_{k+1}] = \frac{\rho \sigma_\theta^2}{\sigma_k^2} (\varepsilon_k - E_{k-1}[\varepsilon_k]).
\]

Recall that \(E_k[\varepsilon_{k+1}] = R_k \cdot \varepsilon_k\) and \(E_{k-1}[\varepsilon_k] = R_{k-1} \cdot \varepsilon_{k-1}\). Substituting in the above equation gives the following recursive relation for the vector \(R_k\):
\[
R_k = \frac{\rho \sigma_\theta^2}{\sigma_k^2} (-R_{k-1}, 1).
\]

Since \(R_1 = \rho \sigma_\theta^2 / \sigma_1^2\), iterating the above recursive relation determines the conditional expectation operator as
\[
R_{k-1}^t = -\frac{(\rho \sigma_\theta^2)^{k-t}}{\sigma_1^2 \ldots \sigma_{k-1}^2},
\]
where \(R_{k-1} = (R_{k-1}^1, \ldots, R_{k-1}^{k-1})\).

The conditional variances are determined following a similar calculation, as follows:
\[
\sigma_{k+1}^2 = \text{var}(\varepsilon_{k+1}) = \text{var}(\varepsilon_{k+1} - E_k[\varepsilon_{k+1}])
\]
\[
= \text{var}(\varepsilon_{k+1} - \frac{\rho \sigma_\theta^2}{\sigma_k^2} (\varepsilon_k - E_{k-1}[\varepsilon_k]))
\]
\[
= \text{var}(\varepsilon_{k+1}) - 2 \frac{\rho \sigma_\theta^2}{\sigma_k^2} \text{cov}(\varepsilon_{k+1}, \varepsilon_k) + \left(\frac{\sigma_\theta^2}{\sigma_k^2}\right)^2 \sigma_k^2
\]
\[
= \text{var}(\varepsilon_{k+1}) - \frac{\sigma_\theta^4}{\sigma_k^2},
\]
Thus, the conditional variances are determined by the recursive relations

\[
\sigma_1^2 = \sigma_0^2 + 2\sigma_0^2 \\
\sigma_{k+1}^2 = \sigma_0^2 + 2\sigma_0^2 - \frac{\sigma_0^4}{\sigma_k^2}.
\]

(47)  
(48)

Note that the posterior variances rapidly decrease and converge to a limit value

\[
\sigma_\infty = \frac{1}{2} \left( \sigma_0^2 + 2\sigma_0^2 + \sqrt{\sigma_0^4 + 4\sigma_0^2\sigma_0^2} \right).
\]

(49)

Appendix C: Proofs

**Proof of Proposition 2.** First, I show that any renegotiation offer that gives the optimal last period incentive and is acceptable to the agent is linear in the performance measures from an ex-ante perspective. At renegotiation time, the principal is restricted to offer a linear contract \( \tilde{c}_{RN} = \alpha_0^{RN}(\tilde{x}_{N-1}) + \beta_N^{RN}(\tilde{x}_{N-1})\tilde{x}_N \), whose coefficients may depend on the first \( N - 1 \) performance measures. The period \( N \) incentive \( \beta_N^{RN} \) does not depend on \( \tilde{x}_{N-1} \) since it is determined only by the conditional variance \( \text{var}(\tilde{x}_N|\tilde{x}_{N-1}) \) which is independent of the actual values of \( \tilde{x}_{N-1} \). Furthermore, from the participation constraint at renegotiation time it follows that

\[
\text{ACE}(\tilde{c}^{IN}, a_N|\tilde{x}_{N-1}, \tilde{a}_{N-1}) = E[\tilde{c}^{IN}|x_{N-1}, a_{N-1}, a_N] - \frac{1}{2} r \text{var}(\tilde{c}^I|\tilde{x}_{N-1}) - \frac{1}{2} \sigma_N^2 \\
\leq \text{ACE}(\tilde{c}^{RN}, a_N|\tilde{x}_{N-1}, \tilde{a}_{N-1}) = \alpha_0^{RN}(\tilde{x}_{N-1}, \tilde{a}_{N-1}) + \beta_N^{RN}E[\tilde{x}_N|\tilde{x}_{N-1}, \tilde{a}_{N-1}, a_N] \\
- \frac{1}{2} r \text{var}(\tilde{c}^{RN}|\tilde{x}_{N-1}) - \frac{1}{2} \sigma_N^2.
\]

(50)

Since the initial contract is linear in \( x_1, \ldots, x_N \) and since the conditional mean of the last period performance measure \( E[\tilde{x}_N|\tilde{x}_{N-1}] \) is linear in \( x_1, \ldots, x_{N-1} \) (due to the joint normality of the distributions), it follows, solving the equation implied by assuming the participation constraint(50) to
be binding for $\alpha_0^{RN}(x_{N-1})$, that $\alpha_0^{RN}$ is linear in $x_{N-1}$,

$$\alpha_0^{RN} = E[c^{IN}|x_{N-1}, a_{N-1}, a_N] - \beta_N^{RN} E[\tilde{e}_N|x_{N-1}, a_{N-1}, a_N]$$

$$- \frac{1}{2} r [\text{var}(c^{IN}|x_{N-1}) - \text{var}(c^{RN}|x_{N-1})]. \quad (51)$$

It then follows that, conditional on being accepted by the agent, the renegotiation offer is of the following form

$$\tilde{c}^{RN} = E[c^{IN}|x_{N-1}, a_{N-1}, a_N] + \beta_N^{RN} E[\tilde{e}_N|x_{N-1}, a_{N-1}, a_N]$$

$$- \frac{1}{2} r [\text{var}(c^{IN}|x_{N-1}) - \text{var}(\tilde{c}^{RN}|x_{N-1})] = c^{IN} + (\beta_N^{RN} - \beta_N^{IN}) E[\tilde{e}_N|x_{N-1}, a_{N-1}, a_N]$$

$$- \frac{1}{2} r [\text{var}(c^{IN}|x_{N-1}) - \text{var}(\tilde{c}^{RN}|x_{N-1})]. \quad (52)$$

From the above equation it follows that $\text{ACE}(\tilde{c}^{RN}) = \text{ACE}(c^{IN})$ at the start of period $N-1$, and that $\tilde{c}^{RN}$ is acceptable at the start of period $N-1$ if, and only if, $c^{IN}$ is acceptable. Thus, $\tilde{c}^{RN}$ is accepted by the agent if offered as a period $N-1$ renegotiation offer. (Note that, in equilibrium, $c^{IN}$ is the outcome of renegotiation from period $N-1$.) Since $\tilde{c}^{RN}$ is an optimal renegotiation offer given $c^{IN}$, and since the efficient period $N$ incentive is independent of $x_{N-1}, a_{N-1}$, the principal has no incentive to renegotiate $\tilde{c}^{RN}$ if it is the initial contract in period $N-1$. This proves that $\tilde{c}^{R}$ is renegotiation-proof in period $N$ and it can replace $c^{RN}$ in period $N-1$.

It remains to show that $\tilde{c}^{RN}$ induces the same actions as $(c^{IN}, \tilde{c}^{RN})$, since the payments to the agent and the principal’s surplus are uniquely determined by the agent’s actions. The period $N$ action is determined by $c^{RN}$ in both cases, since that is the contract in effect at the time the agent provides effort $a_N$. Since $(\tilde{c}^{IN-1}, \tilde{c}^{RN}, a_{N-1}, a_N)$ is an equilibrium in the last two periods, the period $N-1$ action is chosen in anticipation of the period $N$ renegotiation, and so is determined by the (linear) incentive contained in $\alpha_0^{RN}(x_{N-1})$. The reason is that it is suboptimal for the agent to select an action different from $a_{N-1}$ and then reject the renegotiation offer. Thus, the agent’s period $N-1$ action is determined only by the incentives in $\tilde{c}^{RN}$, and so is the same when $\tilde{c}^{RN}$ is the period $N-1$ renegotiation offer.
In the same way, it is shown by backwards induction that $\tilde{c}^{RN}$ can be offered as a renegotiation offer in periods $N-2, \ldots, 2$, and as an initial contract $\tilde{c}^{I1}$ at the start of the first period. □

**Proof of Proposition 3.** At the time the agent selects action $a_t$ at the start of period $t$, his certainty equivalent of compensation given conjectured future actions $\hat{a}_{t+1}, \ldots, \hat{a}_N$ is

\[
ACE(\tilde{c}, \hat{a}_{t+1}, \ldots, \hat{a}_N \mid \tilde{x}_{t-1}, \tilde{a}_{t-1}, a_t) = \alpha_0 + \beta_1 x_t + \cdots + \beta_{t-1} x_{t-1} + \beta_t E[\tilde{x}_t \mid \tilde{x}_{t-1}, \tilde{a}_{t-1}, a_t] \\
+ \beta_{t+1} E[\tilde{x}_{t+1} \mid \tilde{x}_{t-1}, \tilde{a}_{t-1}, a_t, \hat{a}_{t+1}] + \cdots + \beta_N E[\tilde{x}_N \mid \tilde{x}_{t-1}, \tilde{a}_{t-1}, a_t, \hat{a}_N] \\
- \frac{1}{2} \text{var}(\tilde{c} \mid \tilde{x}_{t-1}) - \frac{1}{2} (\hat{a}_t^2 + \hat{a}_{t+1}^2 + \cdots + \hat{a}_N^2) .
\]

Note that, at the time the agent chooses action $a_t$, effort in the previous periods is sunk, and does not enter the agent’s calculations. Furthermore, the fixed portion of the agent’s compensation $\alpha_0 + \beta_1 x_t + \cdots + \beta_{t-1} x_{t-1}$, the variable payments in future periods $\beta_{t+1} E[\tilde{x}_{t+1}] + \cdots + \beta_N E[\tilde{x}_N]$, the risk premium for the variance of the contract, and the cost of future actions do not depend on the choice of period $t$ action. It then follows that the only part of the agent’s certainty equivalent (53) that depends on $a_t$ is

\[
\beta_t E[\tilde{x}_t \mid \tilde{x}_{t-1}, \tilde{a}_{t-1}, a_t] - \frac{1}{2} \hat{a}_t^2 \\
= \beta_t (m_t a_t + \rho e'_t (t-1) \Sigma_{t-1}^{-1} (\tilde{x}_{t-1} - \tilde{w}_{t-1} \tilde{a}_{t-1})') - \frac{1}{2} a_t^2 .
\]

The first-order condition for the incentive compatibility constraint is

\[
\frac{\partial}{\partial a_t} ACE(\tilde{c}, \hat{a}_{t+1}, \ldots, \hat{a}_N \mid \tilde{x}_{t-1}, \tilde{a}_{t-1}, a_t) = m_t \beta_t - a_t = 0 .
\]

It follows that the action induced by the contract is

\[
\hat{a}_t = m_t \beta_t .
\]

As in the full commitment case, the principal’s expected surplus, $U^p$ is the gross benefit $b_1 a_1 +
\[ \cdots + b_N a_N \] less compensation for the agent’s effort \( \frac{1}{2}(\dot{a}_1^2 + \cdots + \dot{a}_N^2) \) and a risk premium for the variance the performance measures \( \frac{1}{2} r \text{var}(\beta_1 \ddot{x}_1 + \cdots + \beta_N \ddot{x}_N) \). \( \square \)

**Proof of Lemma 1.** First, note that \( E_{t-1} E_{t-1} = E_{t-1} \) for all \( 1 \leq t \leq k \). Therefore, the first part follows easily:

\[
E_{t-1} [x_k - E_{t-1} [x_k]] = E_{t-1} [x_k] - E_{t-1} E_{k-1} [x_k] = E_{t-1} [x_k] - E_{t-1} [x_k] = 0.
\]

For the second part, I use the characterization of the conditional variance in terms of conditional expectations \( \text{var}_t(x) = E_t [(x - E_t[x])^2] \).

\[
\text{var}_{t-1} (x_k - E_{k-1} [x_k]) = E_{t-1} [(x_k - E_{k-1} [x_k])^2] = E_{t-1} E_{k-1} [(x_k - E_{k-1} [x_k])^2] = E_{t-1} \text{var}_{k-1} (x_k - E_{k-1} [x_k]) = \text{var}_{k-1} (x_k - E_{k-1} [x_k]).
\]

For the third part, I first prove that \( \text{cov}_{t-1}(x_t, x_k - E_{k-1} [x_k]) = 0 \) for any \( t + 1 \leq k \):

\[
\text{cov}_{t-1}(x_t, x_k - E_{k-1} [x_k]) = E_{t-1} [x_t (x_k - E_{k-1} [x_k])] = E_{t-1} E_t [x_t (x_k - E_{k-1} [x_k])] = E_{t-1} [x_t E_t[x_k] - E_t E_{k-1} [x_k]] = E_{t-1} [x_t (E_t[x_k] - E_t [x_k])] = 0.
\]

Finally, for any \( t + 1 \leq k \leq l \), a similar argument to the above gives

\[
\text{cov}_{t-1}(x_k - E_{k-1} [x_k], x_l - E_{l-1} [x_l]) = E_{t-1} [(x_k - E_{k-1} [x_k])(x_l - E_{l-1} [x_l])] = E_{t-1} E_k [(x_k - E_{k-1} [x_k])(x_l - E_{l-1} [x_l])] = E_{t-1} [(x_k - E_{k-1} [x_k])E_k[x_l] - E_{l-1} E_k [x_l]] = 0.
\]

Probability theory references for the various facts used in the proof are in Malliavin [19], p.184 for the conditional expectation operators and Shiryaev [25], p.83 for the conditional variance. \( \square \)

**Proof of Lemma 2.** The proof is by backwards induction. For \( t = N \), there is nothing to
prove since \( \text{ACE}_{N-1}(w_N) = \text{ACE}_{N-1}(c_N) \) and \( \alpha_N \) is set by the reservation wage constraint
\[
\alpha_N = -\beta N E_{N-1}[\mathbf{x}_{N-1}, \hat{\mathbf{a}}_{N-1}, \hat{\mathbf{a}}_N] + \frac{1}{2} \hat{\mathbf{a}}_N^2 + \frac{1}{2} r \text{var}_{N-1}(c_N).
\]

Now assume that \( \text{ACE}_{k-1}(w_k) = E_{k-1}[\mathbf{c}_k] + \frac{1}{2} \hat{\mathbf{a}}_k^2 + \frac{1}{2} r \text{var}_{k-1}(c_k) \) for all \( t + 1 \leq k \leq N \), and that \( \alpha_k = -\beta_k E_{k-1}[\mathbf{x}_k] + \frac{1}{2} \hat{\mathbf{a}}_k^2 + \frac{1}{2} r \text{var}_{k-1}(c_k) \). I need to show that \( \text{ACE}_{t-1}(w_t) = E_{t-1}[c_t] - \frac{1}{2} \hat{\mathbf{a}}_t^2 - \frac{1}{2} r \text{var}_{t-1}(c_t) = 0 \). By definition,
\[
\text{ACE}_{t-1}(w_t) = E_{t-1}[c_t] - \frac{1}{2} \hat{\mathbf{a}}_t^2 + \cdots + E_{t-1}[c_N] - \frac{1}{2} \hat{\mathbf{a}}_N^2 - \frac{1}{2} r \text{var}_{t-1}(c_t + \cdots + c_N). \tag{57}
\]

Substituting \( \alpha_k \) in equation \( c_k \) for \( t + 1 \leq k \leq N \), I obtain
\[
c_k = \beta_k (x_k - E_{k-1}[x_k]) + \frac{1}{2} \hat{\mathbf{a}}_k^2 + \frac{1}{2} r \text{var}_{k-1}(c_k). \tag{58}
\]

Further substituting in (57) gives
\[
\text{ACE}_{t-1}(w_t) = E_{t-1}[c_t] - \frac{1}{2} \hat{\mathbf{a}}_t^2 + \beta_{t+1} E_{t-1}[x_{t+1}] + \cdots + \beta_N E_{t-1}[x_N - E_{N-1}[x_N]] + \frac{1}{2} r [\text{var}_t(c_{t+1}) + \cdots + \text{var}_{N-1}(c_N)] - \frac{1}{2} r \text{var}_{t-1}(c_t + \cdots + c_N). \tag{59}
\]

First, \( E_{t-1}[x_k - E_{k-1}[x_k]] = 0 \), for \( t + 1 \leq k \leq N \). Second, it follows from Lemma 1 and equation (58) that \( c_t, \ldots, c_N \) are independent at the start of period \( t \), and as a consequence
\[
\text{var}_{t-1}(c_t + \cdots + c_N) = \text{var}_{t-1}(c_t) + \cdots + \text{var}_{t-1}(c_N). \tag{60}
\]

Finally, \( \text{var}_{t-1}(c_k) = \text{var}_{k-1}(c_k) \) for \( t + 1 \leq k \leq N \). Making all the above substitutions in equation (59) proves the desired result:
\[
\text{ACE}_{t-1}(w_t) = E_{t-1}[c_t] - \frac{1}{2} \hat{\mathbf{a}}_t^2 - \frac{1}{2} r \text{var}_{t-1}(c_t). \tag{61}
\]
Proof of Proposition 4. From equation (10) in Lemma 2 it follows that, at the time the agent selects his action $a_t$, his certainty equivalent of future wages is given by

$$\text{ACE}_{t-1}(w_t) = E_{t-1} \left[ \beta_t \left( x_t - \hat{E}_{t-1}[x_t] \right) + \frac{1}{2}(a_t^2 - a_t^2) \right] + E_{t-1} \left[ \beta_{t+1} \left( x_{t+1} - \hat{E}_t[x_{t+1}] \right) + \cdots + \beta_N \left( x_N - \hat{E}_{N-1}[x_N] \right) \right].$$

(62)

Differentiating with respect to $a_t$ gives the first-order condition that determines $a_t$:

$$\frac{\partial}{\partial a_t} \text{ACE}_{t-1}(w_t) = -a_t + \beta_t m_t - \beta_{t+1} \frac{\partial}{\partial a_t} E_{t-1} \hat{E}_t[x_{t+1}] - \cdots - \beta_N \frac{\partial}{\partial a_t} E_{t-1} \hat{E}_{N-1}[x_N] = 0. \quad (63)$$

The action induced by the explicit incentive $\beta_t$ and by the anticipated future incentives is

$$a_t = \beta_t m_t - \sum_{k=t+1}^{N} \beta_k \frac{\partial}{\partial a_t} E_{t-1} \hat{E}_{k-1}[x_k]. \quad (64)$$

Recall that $\hat{E}_{k-1}[x_k] = m_k a_k + R_{k-1} \cdot (\hat{x}_{k-1} - m_{k-1} \hat{a}_{k-1})$, $E_{t-1}[x_t]$ does not depend on $a_t$ for $t + 1 \leq k$, and $E_{t-1}[x_t] = m_t a_t + R_{t-1} \cdot (\hat{x}_{t-1} - m_{t-1} \hat{a}_{t-1})$. It follows that, for any $t + 1 \leq k$,

$$\frac{\partial}{\partial a_t} E_{t-1} \hat{E}_{k-1}[x_k] = m_t R_{k-1}^t, \quad (65)$$

where $R_{k-1} = (R_{k-1}^1, \ldots, R_{k-1}^t, \ldots, R_{k-1}^{k-1})$. Substituting back in the first-order condition for $a_t$ yields

$$a_t = m_t \beta_t - m_t \sum_{k=t+1}^{N} \beta_k R_{k-1}^t. \quad (66)$$

I can use now equations (10) and (66) to substitute in the principal’s problem for the action induced
by $\beta_t$. The first-order condition is
\[
\frac{\partial}{\partial \beta_t} E_{t-1} [(\tilde{z}_t - \tilde{c}_t) + \cdots + (\tilde{z}_N - \tilde{c}_N)] = \frac{\partial}{\partial \beta_t} E_{t-1} [\tilde{z}_t - \tilde{c}_t] = \frac{\partial}{\partial \beta_t} E_{t-1} \left[ \tilde{z}_t + \beta E_{t-1}[\tilde{x}_t] - \frac{1}{2} a_t^2 - \frac{1}{2} r \beta_t^2 \text{var}_{t-1}(\tilde{x}_t) - \beta_t \tilde{x}_t \right] = \frac{\partial}{\partial \beta_t} E_{t-1} \left[ \tilde{z}_t - \frac{1}{2} a_t^2 - \frac{1}{2} r \beta_t^2 \sigma_t^2 \right] = b_t \frac{\partial a_t}{\partial \beta_t} - a_t \frac{\partial a_t}{\partial \beta_t} - r \beta_t \sigma_t^2 = 0.
\]
Since $\frac{\partial a_t}{\partial \beta_t} = m_t$, it follows that $r \beta_t \sigma_t^2 = m_t (b_t - a_t)$ . Solving for $a_t$, I obtain
\[
a_t = b_t - \frac{r \beta_t \sigma_t^2}{m_t}.
\]
Substituting $a_t$ in equation (66) and solving for the optimal incentive gives
\[
\beta_t = \frac{m_t b_t}{m_t^2 + r \sigma_t^2} + \frac{m_t^2 \sum_{k=t+1}^{N} \beta_k R_{k-1}^t}{m_t^2 + r \sigma_t^2}.
\]
\[\square\]

**Proof of Proposition 5.** It is sufficient to show that the optimal actions induced by the sequence of contracts in Proposition 4 are the same as the actions induced by the renegotiation-proof contract described in Proposition 3. From (6) and Lemma 2, the principal’s surplus is the same in both cases, in that the agent is only compensated for his effort and a risk premium for the posterior variance of each performance measure. Since the payoffs are uniquely determined by the induced actions it follows that, from an ex-ante (start of the first period) perspective, the contract $\tilde{c} = \tilde{c}_1 + \cdots + \tilde{c}_N$ is the same as the renegotiation-proof contract described in Proposition 3.

The proof that both types of contracts induce the same actions is by backwards induction, starting with the last period. Under commitment to fairness, as with renegotiation, the period $N$ action and the period $N$ incentive do not depend on the previous actions, or on the specific value of previous performance measures. The reason, as before, is that when contracting at the start of period $N$, the role played by the available information is to reduce the variance in the period $N$ performance measure, and that variance does not depend on either the specific values of
the previous performance measures or the previous actions. It follows that the principal’s optimal choice of risk for the agent when setting the period $N$ incentive does not depend on either the specific value of the previous performance measures or the previous actions. Thus, the principal’s problem in setting the incentive for period $N$ is the same in both cases, and as a result the induced period $N$ action is the same.

Once the period $t$ incentive rate is set, the fixed wage is given by the fairness restriction. Then, the period $t$ fixed wage depends linearly on $x_{t-1}$ because $E[\tilde{x}_t|x_{t-1}, \tilde{y}_{t-1}]$ is linear in $x_{t-1}$. From an ex-ante perspective, the period $t$ contract contains a (random) linear term in $\tilde{x}_{t-1}$, which contributes in addition to the incentives from the period $t-1$ contract to the agent’s choice of action in period $t-1$. At the start of period $t-1$, the agent’s cumulative future compensation $\tilde{c}_{t-1} + \cdots + \tilde{c}_N$ is linear in $\tilde{x}_{t-1}, \ldots, \tilde{x}_N$, while the induced actions $a_{t-1}, \ldots, a_N$ are the same as in the renegotiation case. Both the principal and the agent anticipate future contracts $\tilde{c}_t, \ldots, \tilde{c}_N$ and have a common conjecture regarding their terms. It follows that the principal’s problem is the same as in the renegotiation case and that the total incentive for the period $t-1$ action is the same. The main difference is that, with commitment to fairness, the period $t-1$ incentives are split between the period $t$ variable wage and the period $t, \ldots, N$ fixed wages. The agent’s actions are the same and the principal’s surplus is the same in both cases. □

**Proof of Proposition 6.** From Proposition 4, I have that

$$\beta_t = \frac{m_t b_t + m_t^2 \sum_{k=t+1}^{N} \beta_k R_{k-1}^t}{m_t^2 + r \sigma_t^2} = \frac{m_t b_t + m_t^2 \beta_{t+1} R_t^t + m_t^2 \sum_{k=t+2}^{N} \beta_k R_{k-1}^t}{m_t^2 + r \sigma_t^2},$$

(70)

and that, for $\beta_{t+1}$, I have

$$\beta_{t+1} = \frac{m_{t+1} b_{t+1} + m_{t+1}^2 \sum_{k=t+1}^{N} \beta_k R_{k-1}^{t+1}}{m_{t+1}^2 + r \sigma_{t+1}^2}.$$

(71)
Solving for the second term in the numerator in (71) gives

\[
\sum_{k=t+2}^{N} \beta_k R_{k-1}^{t+1} = -\frac{b_{t+1}}{m_{t+1}} + \left(1 + r \frac{\sigma_{t+1}^2}{m_{t+1}^2}\right) \beta_{t+1}.
\]  
(72)

For information system \(\eta^1\),

\[
R_{k-1}^t = \frac{\sigma_\theta^2}{(k-1)\sigma_\theta^2 + \sigma_\delta^2}
\]

\[
\sigma_t^2 = \sigma_\delta^2 \left(1 + \frac{\sigma_\theta^2}{(k-1)\sigma_\theta^2 + \sigma_\delta^2}\right).
\]
(73)

It follows that \(R_{k-1}^t = R_{k-1}^{t+1}\) for \(t + 1 \leq k - 1 \leq N - 1\) and \(R_t^t = \sigma_t^2/\sigma_\delta^2 - 1\). Substituting then in (70) using (72) and (73) gives (20). Finally, I substitute \(a_t = b_t - r\beta_t \sigma_t^2/m_t\) and \(\beta_{t+1} = m_{t+1}(b_{t+1} - a_{t+1})/(r\sigma_{t+1}^2)\) in (20) to obtain (21).

For information system \(\eta^2\), I have

\[
R_{k-1}^t = -\frac{(-\rho \sigma_\theta^2)^{k-t}}{\sigma_t^2 \ldots \sigma_{k-1}^2}.
\]
(74)

It follows that \(R_{k-1}^t = (-\rho \sigma_\theta^2/\sigma_t^2)R_{k-1}^{t+1}\) and \(R_t^t = \rho \sigma_\theta^2/\sigma_t^2\). Substituting then in (70) using (72) and (74) gives (22). Finally, I substitute \(a_t = b_t - r\beta_t \sigma_t^2/m_t\) and \(\beta_{t+1} = m_{t+1}(b_{t+1} - a_{t+1})/(r\sigma_{t+1}^2)\) in (22) to obtain (23).

\[\square\]

References


