The Timing of Analysts’ Earnings Forecasts

Ilan Guttman
Stanford University
Graduate School of Business
518 Memorial Way
Stanford, CA 94305
iguttman@stanford.edu

May 18, 2005

---

1I am grateful to Eugene Kandel for his guidance and encouragement. I would also like to thank Sasson Bar-Yosef, Elchanan Ben-Porath, Ohad Kadan, Motty Perry, Madhav Rajan, and seminar participants at Columbia University, Hebrew University, Northwestern University, Stanford University, Tel Aviv University, and Tilburg University for their helpful comments and suggestions.
Abstract

Most of the literature assumes that the order and timing of analysts’ earnings forecasts are determined exogenously. This paper analyzes the equilibrium timing strategies for analysts. Consistent with the prior literature I assume that analysts care foremost about the accuracy of their forecasts, but in some cases may have an incentive to bias their forecasts. The paper further assumes that investors reward early forecasting analysts. The main trade-off that an analyst faces in determining the timing of his forecast is between precision and timeliness. The paper introduces a timing game with two analysts, derives its unique pure strategies Subgame Perfect Equilibrium, and provides empirical predictions. The equilibrium has two patterns: either the times of the analysts’ forecasts cluster, or there is a separation in time of the forecasts. All else equal, an increase in the precision of an analyst’s private signal induces earlier forecast by this analyst, and increases the likelihood that the analysts’ forecasts will cluster in time. This prediction may be used in empirical studies to infer the precision of the analysts’ signals from the observed timing of the forecasts.
1 Introduction

Sell-side analysts are one of the most important sources of information for investors in the stock market. Among other services, they provide early forecasts of firm’s earnings, forecasts which investors use for stock valuation. The analysts’ forecasts are based on information they generate privately as well as on publicly available information, which includes prior forecasts of other analysts. This suggests that every analyst is both a supplier of information to other analysts and a consumer of such information, that comes from other analysts. The degree to which an analyst plays each role is determined by his position in the sequence of forecasts announcements – that is to say the timing of his forecast. The question that generated this paper is whether the timing of analysts’ forecasts is determined exogenously or at random, as implicitly assumed in much of the literature, or whether analysts choose the timing of their forecasts strategically. The answer to this question may yield additional insights into the behavior of sell-side analysts that received so much attention recently. Specifically, the information contained in the timing and order of analysts’ forecasts may help decipher their informational content. Ignoring this information may lead to inconsistent inferences. In this paper I propose a theory for the timing of analysts’ earnings forecasts, and analyze their equilibrium timing and reporting strategies.

I follow the literature in assuming that analysts care primarily about the accuracy of their forecasts (e.g. Mikhail, Walther, Willis [1999], Hong and Kubik [2003]), but in some cases may also have an incentive to bias their forecasts (e.g. Dugar and Nathan [1995], Hong and Kubik [2003], Lim [2001], and Das et al [1988]). Jackson (2005) documents the analyst’s trade off between accuracy and optimistic bias. In a setup where the analysts’ objective function is based only on these two assumptions, all the analysts would optimally make their forecasts immediately before the earnings announcement by the firm, since this is when their forecasts are the most accurate. This is clearly not a reasonable outcome. Investors would be willing to reward a deviating analyst who provides an earlier signal. Hence, there must be an offsetting effect. To capture this effect, I propose an additional component to the analyst’s payoff function. I assume that the compensation of an analyst declines in the precision of the investors’ beliefs about the earnings of the firm at the time of his forecast. Thus, the expected payoff of an

\[1\] In his introduction, Jackson (2005) says: “the analyst must trade off the short-term incentive to lie and generate more trade against the long-term gains from building a good reputation”.
early-reporting analyst is higher than the expected payoff of an analyst who is making the same forecast at a later time. This seems to be a natural assumption. Uninformed investors are willing to pay for information (forecast) that increases the precision of their beliefs. At the extreme case where investors are perfectly informed about the forthcoming earnings of the firm, a forecast is worthless to them, and obviously they are not willing to pay for additional information. This new assumption finds an empirical support in Cooper, Day and Lewis (2001) and in Jackson (2005). Following the incorporation of this new assumption about the analyst’s payoff function, the analyst faces the trade-off between an earlier, but less precise forecast, and a later but more precise one. I further assume that a continuous stream of public information from exogenous sources (other than analysts’ earnings forecasts) arrives over time. As to investors, I assume that they (as well as other analysts) do not necessarily know the actual bias of an analyst, which further complicates their inference.

I start by deriving and analyzing the optimum in a single analyst case (hereafter the unconstrained optimum), which provides the basic intuition and serves as a benchmark. The optimal forecast timing for a single analyst is determined by the precision of his private signal and by his cost of a forecast error. The intuition is straightforward: higher precision induces an earlier forecast, while the higher cost of an error postpones the forecast for the purpose of gaining more information over time. Next, I introduce a timing game with two analysts. The game has a unique Subgame Perfect Equilibrium in pure strategies. The equilibrium takes one of two possible patterns. When the two analysts are sufficiently different from each other, then each publishes his forecast at the respective unconstrained optimal time – the non-clustering pattern. The only other alternative, is when the two analysts issue forecasts one immediately after the other, creating an endogenous clustering in time of forecasts. The likelihood of the clustering equilibrium pattern declines in the distance between the unconstrained optimal timing of the two analysts; increases in the precision of the private signals of the analysts; and increases in

---

2In their introduction, Cooper, Day and Lewis (2001) state: “Since brokerage firms’ profits depend directly on commission revenues, analysts compensation is based, in part, on the trading volume generated by their research. This gives superior analysts an incentive to release information before other analysts in order to capture trading volume for their firms”. Jackson (2005) shows that high reputation analysts generate more trade to their brokerage firm, and that timeliness “may be significantly more important than accuracy in determining an analyst’s ranking”.

3Ivkovic and Jagadeesh (2004) find that the informational content of analysts’ earnings forecast revisions, generally increases over event time.
the precision of the investors’ beliefs about the bias of the analysts. At times of extensive arrival of new information (around an exogenous event), the model predicts an endogenous clustering of timings of analysts’ forecasts that are very common in the data. The model predicts the order, the timing, and the reported forecasts of both analysts.\footnote{The model can be applied to analysts’ target prices and earnings growth estimates in a straightforward way. I conjecture that a similar argument can be made about stock recommendations; however, since those are on a discrete grid, a different methodology must be employed.} The paper provides a complete analysis of the two–analysts case, and discusses a possible extension to a multi–analysts setup.

While the literature on the incentives of forecasters, and in particular analysts, is quite extensive, very little has been said about the order and timing of analysts’ forecasts. To the best of my knowledge, the only related theoretical paper that addresses the endogenous timing of forecasters is Gul and Lundholm (1995). They present a model of two agents, where each has to choose the timing of his prediction about the future value of a project. Each agent observes a private signal of the project’s value. They assume that the value of the project equals the sum of the two private signals. All else equal, the agents prefer to predict sooner rather than later. Contrary to my model, Gul and Lundholm (1995) show that agents’ forecasts always cluster in time. The setup of my paper is more representative of analysts’ environment and uses less restrictive assumptions. The main differences in the setup are that, contrary to Gul and Lundholm (1995): I do not assume that the sum of the private signals of the analysts equals the earnings of the firm; I assume arrival of exogenous information over time; I assume that the benefit from the forecast depends on the precision of the investors’ beliefs, and I explicitly model the possibility that forecasts may be biased.

Several empirical papers are of particular relevance: Cooper, Day and Lewis (2001) find that lead analysts, identified by their measure of forecast timeliness, have a greater impact on stock prices than follower analysts. Further, they find that performance rankings based on forecast timeliness are more informative than rankings based on abnormal trading volume and forecast accuracy. Lin, McNichols, and O’Brien (2003) provide evidence that analysts’ affiliation influence their timeliness in downgrading their recommendations. Bernhardt and Campello (2004) study the relation between the forecast and its timing, but their focus is mostly whether firms manage analysts’ forecasts (expectation management) and on the forecast revisions towards the end of the forecasting period.
There is an extensive literature claiming that analysts may have incentives to bias their forecasts and recommendations.\(^5\) Dugar and Nathan (1995) show that financial analysts of brokerage firms that provide investment banking services to a company are optimistic, relative to other analysts, in their earnings forecasts and stock recommendations. Lin and McNichols (1998) find that lead and co-underwriter analysts’ growth forecasts and recommendations are significantly more favorable than those made by unaffiliated analysts, although their earnings forecasts are not generally greater. Michaely and Womack (1999) document that analysts may be too optimistic about firms from which they are trying to solicit underwriting business. Jackson (2005) shows that overoptimistic forecasts can be due to trade generation incentives. Hong and Kubik (2003) show that career concerns may induce overoptimistic forecasts. Bernhardt and Campello (2004) attribute the bias in the analysts’ forecasts to expectation management by the managers, who try to avoid negative earnings surprises.\(^6\) Lim (2001) claims that an analyst’s forecast bias is fully rational because it induces the firm’s management to produce better information to optimistic analysts.\(^7\) There is an extensive literature (both theoretical and empirical) that examines the relationship between reputational concerns and herding behavior (e.g., Scharfstein and Stein (1990), Trueman (1994), and Welch (2000)). In these models, the reputation arises from learning over time about agent’s exogenous characteristics (e.g. ability) through his observed behavior. Considerations for reputation or “career concerns” can lead agents to underweight (or even ignore) private information, and to herd. I use a reduced form for the analyst’s objective function, where forecasting errors induce reputation costs. The origin of the reputation costs is not modeled directly; rather it is assumed to be given exogenously.

The rest of the paper is organized as follows: Section 2 presents the setup of the model. Section 3 derives the optimal analyst’s behavior in a single analyst case. Section 4 presents and discusses the timing game between two analysts. Section 5 concludes.

\(^5\) A very strong implicit testimony, is the Global Settlement (April 2003) in which the largest investment banks agreed to pay $1.4 billion in fines and reparations for potentially misleading investors in their analysts’ reports. For details about the settlement see http://www.oag.state.ny.us/press/statements/global_resolution.html.


\(^7\) Irvine (2003) asserts that an analyst’s coverage of a firm induces higher commissions to his brokerage firm; nevertheless, analysts can not induce extra commissions by simply biasing their published forecast.
2 Model Setup

Most of the literature implicitly assumes that the timing of analysts’ earnings forecasts is random or exogenously determined. The main objective of analysts is assumed to be the accuracy of their forecasts, i.e., to minimize the expected squared error of their forecasts. However, as discussed in the introduction, analysts may have other incentives that may bias their forecasts. The incentives of analysts to bias their forecasts at a specific time (e.g., quarter) are not transparent and not perfectly known to investors. The model assumes that the actual bias in analysts’ forecast may be unknown to the investors and to the other analysts (this is similar to the assumption of Fischer and Verrecchia (2000) about managers’ reporting bias). A known bias is a particular case (which is equivalent to an unbiased analysts’ case). For the simplicity of the disposition, I assume that the analyst’s expected utility is linear in his bias (the linear coefficient of analyst $i$ is denoted by $\alpha_i$). Nevertheless, the model is robust to a large class of functional dependence between the analyst’s bias and his expected payoff, and is not restricted to linear functional dependence (later the paper will elaborate on this and define the larger class of functional dependence).

The above two components of the analyst’s utility function (precision/reputation and bias incentives) are prevalent in the literature. In a conventional model based solely on these two components, all analysts would optimally forecast immediately before the earnings announcement of the firm, since this is when their forecasts are the most accurate. This is clearly not a reasonable outcome, because investors would be willing to reward a deviating analyst who provides early forecast. So, there must be an offsetting effect that is ignored. The additional assumption that I propose in order to capture this offsetting effect is the following: the payoff of an analyst depends on the precision of the investors’ beliefs about the firm’s earnings immediately prior to the analyst’s forecast. The less precise the investors’ beliefs about the firm’s earnings are, the more valuable is the forecast of the analyst to the investors, and hence the higher is the payoff of the analyst (for a given forecast). Following is the motivation for this new assumption.

---

8While minimizing the squared error is the prevalent objective function in the literature, Basu and Markov (2003) argue that analysts’ behavior is rational if we assume that they minimize their absolute forecast error rather than a quadratic cost function.
The analyst is paid by the brokerage house he works for. A big part of the earnings of a brokerage house is from trading commissions from its investors – clients. The brokerage house and the analyst want to maintain existing clients, to attract new clients and to increase the volume of trade executed through the brokerage house. The benefit that investors receive from analysts’ earnings forecasts is early access to information. The most preferred clients of an analyst get his forecast first; only later do the less preferred clients get this forecast, and eventually it is publicly published. Access to the information before it becomes public is so valuable to investors that they are willing to pay for it. The investors use this information in order to form beliefs upon which they make their financial decisions. These decisions eventually generate trade in the stock. In the extreme case, where the investors are perfectly informed about the future earnings of the firm, an analyst’s forecast is worthless to investors. Moreover, in this case, an analyst’s forecast will not generate any trade in the stock. The less informed investors are – that is, the lower the precision of their beliefs about the firm’s earnings, the more valuable the analyst’s forecast is to investors and the higher the trade it may generate. Jackson (2005) finds that the reputation of an analyst increases in the timeliness of his forecasts, which provides additional motivation to the assumption that the expected payoff of an analyst decreases in the time of his forecast.

In financial markets, as time advances, more public information about the forthcoming earnings arrives. This information arrives from analysts’ forecasts as well as from many other relevant sources of information (Macro economics, competitors, conference calls etc.). I refer to all information other than analysts’ forecasts as exogenous information. I denote the precision of the investors’ beliefs about the earnings of the firm at time \( t \) by \( f(t) \). I assume that the arrival of the exogenous information is continuous, meaning that the precision of the investors’ beliefs is continuously increasing in time (in all times except at a time of a forecast publication, where there will be a discrete increase in the precision of the investors’ beliefs).\(^9\)

All else equal (including the precision of the forecast), the sooner the analyst provides his forecast, the more valuable his forecast is to his clients and the more trade it may generate – hence his expected payoff is higher. But there is also a cost for early forecasting. The sooner

\(^9\)The model is robust to any process of information arrival – including discontinuous processes. Nevertheless, the continuity assumption simplifies the analysis and the disposition by making the analysts’ utility function differentiable with respect to time.
the analyst publishes his forecast, the less accurate is the public information he uses to generate his forecast; hence, his expected forecast error is higher. The basic trade-off that analysts face in determining the timing of their forecasts is between timeliness and precision. Cooper, Day and Lewis (2001) point out (empirically) the willingness of lead analysts to trade accuracy for timeliness due to their desire to maximize compensation.

The model assumes that an analyst has to publish his forecast at some point during the “forecasting season” $t \in [0, T]$, e.g., between the earnings report of the previous period (quarter) and the forthcoming earnings report. After the forecasting season, the firm reports its realized earnings, denoted by $\pi$. At the beginning of the forecasting season ($t = 0$), investors are assumed to have normally distributed prior beliefs about the earnings of the firm $- \pi_0 \sim N(\mu_{\pi_0}, \sigma_{\pi_0}^2)$. The precision of the prior beliefs is denoted by $f(0) \equiv 1/\sigma_{\pi_0}^2$. As time progresses, there is a continuous stream of exogenous information that increases the precision of the investors’ beliefs $- f(t)$ (while the beliefs remain normally distributed). For all $t_1 > t_2$ we have $f(t_1) > f(t_2)$, $f(0) > 0$ and $f(T) < \infty$.

While deciding about the earnings forecast, an analyst uses all the available public information as well as his private information about the forthcoming earnings of the firm. I assume that analyst $i$ gets a private signal about the earnings of the firm $- \tilde{\psi}_i = \pi + \tilde{\varepsilon}_i$, where $\tilde{\varepsilon}_i \sim N(0, \sigma_{\tilde{\varepsilon}_i}^2)$ is independent of $\pi$ (in the case of more than one analyst, for all $i \neq j - \tilde{\varepsilon}_i$ is independent of $\tilde{\varepsilon}_j$). I denote the precision of analyst $i$’s private signal by $f_{S_i} \equiv 1/\sigma_{\tilde{\varepsilon}_i}^2$. There exists an extensive literature on how forecasters and in particular analysts decide about the weight they place on private and public information (e.g., Chen and Jiang (2005)). I assume that while forming their beliefs, analysts are rational and apply base rule. As to the time at which an analyst observes his private signal, in the model it does not influence the equilibrium results (as long as it happens before the equilibrium timing of his forecast). For simplicity, let’s assume that the analysts get their private signals at $t = 0$.

I use a reduced form of the analyst’s objective function, which captures the above characteristics and trade-off. The expected utility of analyst $i$ who makes a forecast at time $t$ is assumed

\[10^{th}\text{Assuming that companies manipulate the reported earnings by a constant (see for example Stein (89) and Guttman, Kadan, Kandel (2005)), would not influence the results.}

\[11^{th}\text{It could be an approximation of a discrete process of normally distributed signals.}\]
to be:

\[ EU^i_t = \alpha_i \left( \pi^F_{i,t} - E \left[ \pi | \psi_i, I_t \right] \right) - \beta_i E \left[ \left( \pi^F_{i,t} - \pi \right)^2 | \psi_i, I_t \right] - f(t) \]  

where \( \alpha_i \sim N(\mu_{\alpha_i}, \sigma^2_{\alpha_i}) \) is the forecast bias parameter, \( \beta_i \) is a positive constant, \( \pi^F_{i,t} \) denotes the forecast of analyst \( i \) (published at time \( t \)), \( \psi_i \) is the private signal of the analyst, and \( I_t \) is all the public information available at time \( t \) (immediately prior to the analyst’s forecast) which includes the preceding analysts’ forecasts. The realization of \( \alpha_i \) is known only to the analyst himself, where \( \beta_i \) is common knowledge. \( E \) is the expectation operator.\(^{12}\)

The utility function of the analyst has three components: the term \( \left( \pi^F_{i,t} - E \left[ \pi | \psi_i, I_t \right] \right) \) captures the analyst’s incentives to bias his forecast; The term \( E \left[ \left( \pi^F_{i,t} - \pi \right)^2 | \psi_i, I_t \right] \) is the mean squared error (MSE) of the analyst’s forecast and captures the analyst’s desire to be precise (reputation/precision); and \( f(t) \) captures the incentive of the analyst to provide his forecast at a stage where the precision of the investors’ beliefs is low (timeliness). With out loss of generality the coefficient of \( f(t) \) is normalized to be one.

I believe that the three incentives of analysts mentioned above are the central ones to their behavior. As in every model, there may be other incentives of analysts which are not accounted for in the model.

Note that immediately following the analyst’s forecast, there is a discrete jump in the precision of the investors’ beliefs, and thereafter, until the forecast of the next analyst, the precision of the investors’ beliefs continuously increases according to the exogenous information process.

Using the above setup, I first solve the optimization problem of a single analyst case – his optimal forecasting timing and the optimal forecast at that point in time. Next, I study a game between two analysts, who must decide at what point in time to publish their forecasts. Since the analyst’s expected utility depends on the precision of the public’s information at the time of

\(^{12}\)A more general utility function is: \( EU^i_t = \alpha_i \left( \pi^F_{i,t} - E \left[ \pi | \psi_i, I_t \right] \right) - \beta_i E \left[ \left( \pi^F_{i,t} - \pi \right)^2 | \psi_i, I_t \right] - \gamma_i g \left( f(t) \right) \) where \( g(\cdot) \) is a continuously increasing function. I show in Appendix 1 that the results of the model holds for a very big set of functions \( g(f(t)) \). w.l.o.g. \( \gamma_i \) is normalized to equal 1.

An alternative way for modeling, is to assume a multiplicative utility function rather than an additive one, for example \( EU^i_t = E \left[ \alpha_i \pi^F_{i,t} - \beta_i f(t) \left( \pi^F_{i,t} - \pi^R \right)^2 | \psi, I_t \right] \).

In this case, the optimal forecasting timing is typically a corner solution. Nevertheless, in general, the comparative statics work (weakly) in the same directions as in the paper’s additive utility function.

8
forecast (and not on the time per se), a strategic interaction arises between the analysts, which should be considered. I derive a pure strategies Subgame Perfect Equilibrium of this game and prove its existence and uniqueness.

3 The Single Analyst Case

This section analyzes the case of a single analyst who has to choose the time of his forecast. This simple case is not a strategic game, but rather a simple optimization problem. The analyst has to decide about the time of his forecast and the value to be forecasted. While solving for the optimum I first derive the optimal forecast for every possible forecasting time, then, given the optimal forecasts, I find the optimal timing of the forecast (to be precise, I derive the precision of the investors’ beliefs at which it is optimal for the analyst to publish his forecast).

3.1 The Optimal Forecast

The analyst’s First Order Condition with respect to the forecast at a given time $t$ is

$$\alpha_i - 2\beta_i E \left( \pi_i^F - \pi | \psi_i, I_t \right) = 0.$$ 

The Second Order Condition for maximum is satisfied. Hence, for every given timing of forecast $t$, the analyst’s optimal forecast is:

$$\pi_{i,t}^F = \frac{\alpha_i}{2\beta_i} + E \left( \pi | \psi_i, I_t \right)$$

(2)

where $\mu_{\pi_t} = E \left( \pi | I_t \right)$ and $\sigma^2_{\pi_t} = Var \left( \pi | I_t \right)$ are the mean and variance of investors’ beliefs at time $t$ immediately prior to the forecast respectively (Note that $\mu_{\pi_t}$ may be different from $\mu_{\pi_0}$).

The optimal forecast of the analyst is to bias his forecast by the constant $\frac{\alpha_i}{2\beta_i}$. Although the analyst’s optimal forecast is linear in his private signal, it does not fully reveal his private signal – since $\alpha_i$ is not known to investors. Only in the case where the bias parameter $\alpha_i$ is common knowledge, the analyst’s forecast fully reveals his private signal.
Substituting the optimal forecast of the analyst into his utility function yields:

\[ EU_i^t = \frac{\alpha_i^2}{4\beta_i} - \frac{\beta_i}{f(t) + f_{s_i}} - f(t). \]  \hspace{1cm} (3)

An increase in \( f(t) \) has two opposite effects on the analyst’s expected utility. On the one hand, it increases the analyst’s information and reduces the expected cost of a forecast error (reduces \( \frac{\beta_i}{f(t) + f_{s_i}} \)). On the other hand, the analyst incurs the direct cost of a later forecast. In the next section, I find the analyst’s optimal behavior in solving the above trade-off.

### 3.2 The Optimal Timing of the Forecast

The time per se is not an important factor, rather, what matters is the precision of the investors’ beliefs at each point in time. In light of the above trade-off, in order to find the precision of the investors’ beliefs at the optimal forecasting timing, I take the derivative of the analyst’s expected utility (equation 3) with respect to the precision of the investors’ beliefs.

Optimization yields

\[ \frac{\beta_i}{(f(t) + f_{s_i})^2} - 1 = 0, \]

which suggests that the analyst should forecast at

\[ f(t) = \sqrt{\beta_i} - f_{s_i}. \]

But, the precision of the investors’ beliefs at which the analyst can forecast is constrained by the precision of the investors’ beliefs at the beginning and at the end of the forecasting season, i.e. \( f(0) \) and \( f(T) \).

I denote the analyst’s optimal forecasting time by \( t_i^* \). Incorporating the constraint of the forecasting season, we get:

**Proposition 1**

\[
t_i^* = \begin{cases} 
0 & \text{if } f_{s_i} \geq \sqrt{\beta_i} - f(0) \\
-1/(\sqrt{\beta_i} - f_{s_i}) & \text{if } f(0) < f_{s_i} < \sqrt{\beta_i} - f(0) \\
T & \text{if } f_{s_i} \leq \sqrt{\beta_i} - f(T)
\end{cases}
\]

\[ 13 \text{ EU}_i^t = \alpha_i \frac{\alpha_i}{2\beta_i} - \beta_i E \left[ \left( E[\pi|\psi, I_i] + \frac{\alpha_i}{2\beta_i} - \pi \right)^2 |\psi, I_i \right] - f(t) = \alpha_i \frac{\alpha_i}{2\beta_i} - \frac{\alpha_i^2}{2\beta_i} - \beta_i \text{Var}(\pi|\psi, I_i) - f(t). \]  \hspace{1cm} \text{in the appendix} I \text{ show that } \text{Var}(\pi|\psi, I_i) = \frac{1}{f(t) + f_{s_i}}

\[ 14 \text{ The time value of money could be accounted for as well, without qualitatively affecting the results.} \]
where \( f^{-1} (\sqrt{\beta_i} - f_{S_i}) \) is the time at which the precision of investors’ beliefs is \( f(t) = \sqrt{\beta_i} - f_{S_i} \).\(^{15}\)

Hereafter, I refer to the time \( t_i^* \) as the “unconstrained optimum timing”.

If the analyst’s private signal is sufficiently precise, he will publish his forecast immediately at the beginning of the forecasting period, since an increase in the precision of the public information (and hence in his forecast) is not sufficient to compensate for the associated cost of late forecast. This means that his expected utility monotonically decreases in time, and hence he forecasts as early as he can. This is illustrated by the High Precision case in the figure below. On the other hand, for sufficiently low precision of the analyst’s private signal, the analyst will wait to gain as much public information as he can, and will forecast at \( T \). See the Low Precision case in the figure below. In the “intermediate case”, the trade-off is such that the analyst waits until time \( t_i^* \) where \( f(t_i^*) = \sqrt{\beta_i} - f_{S_i} \). After this time, the cost of late forecasting exceeds his payoff from the increase in the precision of the public information he can use. This implies that for \( \sqrt{\beta_i} - f(T) < f_{S_i} < \sqrt{\beta_i} - f(0) \): the expected utility of the analyst monotonically increases for \( f(t) < f(t^*) \), and after the time \( t^* \) it monotonically decreases. This is illustrated by the Interior solution case in the figure below.

\[^{15}\text{If we assume that the coefficient of the bias in the analyst’s expected payoff, is not a constant and is a function of } f(t) \text{ (i.e. } \alpha_i(f(t)) \text{ and not just } \alpha_i \text{), then the optimal forecast time of the analyst is: } f(t_i^*) = \sqrt{1 - \frac{\alpha_i(f(t))}{\alpha_i(f(0))}} \sqrt{\beta_i} - f_{S_i} \text{. The model is robust to all } \alpha_i(f(t)) \text{ for which the above } f(t_i^*) \text{ is well defined.}\]
Corollary 1 (Comparative Statics) Given the interior solution for $t^*_i$, i.e. $\sqrt{\beta_i} - f(T) < f_{S_i} < \sqrt{\beta_i} - f(0)$, the optimal forecasting time of the analyst decreases in the precision of his private signal ($f_{S_i}$); and increases in his error/reputation cost parameter ($\beta_i$).

The above corollary is quite intuitive. Less precise private signal of the analyst induces him to postpone his forecast and gain from the increased precision of the public’s information. On the other hand, the lower his reputation cost for a given forecast error (captured by $\beta_i$), the higher his propensity to risk a large error in order to provide his forecast at an early stage. Since the coefficient of the precision of the investors’ beliefs in the analyst’s utility function is normalized to equal 1, higher rewards for early forecast is equivalent to reducing both $\alpha_i$ and $\beta_i$, and induces an earlier forecast.

The timing of the analyst’s forecast is independent of the precision of the investors’ beliefs about his bias. The realizations of both the private signal and the bias parameter do not affect the optimal forecasting timing. The realized signal does not influence the precision of the analyst’s beliefs nor the precision of the investors’ beliefs following his forecast (it influences only the conditional expectations but not the conditional variance). Hence, the realized signal does not influence the trade-off that the analyst faces while choosing the time of his forecast. As to the value of $\alpha_i$, changes in $\alpha_i$ linearly change the bias in the analyst’s forecast. All else equal, a change in $\alpha_i$ affects only the first expression in (3) which is independent of $f(t)$, and is outside the analyst’s timing trade-off decision. From the investors’ perspective, the closer the realized value of $\alpha_i$ to its mean, the more accurate their beliefs following the analyst’s forecast are. The more confident the investors are regarding the analyst’s bias, the more they learn on average from his forecast, i.e. the higher the weight they attach to the forecast.

In the next section I introduce a timing game between two analysts. Due to the strategic interaction between the analysts, both their prior beliefs about the bias of the other analyst and the precision of the private signals will influence their equilibrium strategies.

4 Timing Game With Two Analysts

Most stocks are covered by more than one sell-side analyst. This implies that it is important to understand how competition alters the behavior of sell-side analysts. Recall that following
an analyst’s forecast, there is a discrete increase (“jump”) in the precision of investors’ beliefs. Hence, the support of the precision of investors’ beliefs at which an analyst can publish his forecast is no longer the entire interval \([f(0), f(T)]\), as it was in the single analyst case. In the context of a two-analysts’ game, one can think of two different time lines: the calendar time line and the precision of the investors’ beliefs time line. While the calendar time line is continuous, the precision of the investors’ beliefs time line has a “jump” at the time of an analyst’s forecasts. Figure 1 presents the two time lines for analyst \(i\). The horizontal axes (the calendar time line) obtains continuous values, but on the vertical axes (the precision of investors’ beliefs at which analyst \(i\) can publish his forecast) there is a discrete jump ("hole") following the forecast of analyst \(j\) at time \(t_0\). Note that the location of this "hole" is determined endogenously, according to the equilibrium timing strategies of the analysts. The size of the discrete jump following the forecast of analyst \(j\) is denoted by \(\Delta f_{Sj}\).

While determining the optimal forecasting timing, the analyst cares about the precision of the investors’ beliefs, and not on the time per se. The fact that a forecast of an analyst changes the precision of the investors’ beliefs at which the other analyst can publish his forecast immediately introduces strategic interaction between the two analysts. Let’s assume, for example, that the unconstrained optimal forecasting time for analyst \(i\) is \(t^*_i\). If he waits till \(t^*_i\), he bears the risk that analyst \(j\) will step in front of him and forecast at \(t^*_j - \varepsilon\). If analyst \(j\) does forecast at \(t^*_j - \varepsilon\), analyst \(i\) will face investors’ beliefs with precision of \((f(t^*_i - \varepsilon)) + \Delta f_{Sj}\) (where \(\Delta f_{Sj}\) denotes the increase in the precision of the investors’ beliefs due to the forecast of analyst \(j\)). This will decrease the expected utility of analyst \(i\) relative to forecasting right before analyst \(j\). Analyst \(j\) has to take into account that analyst \(i\) may hence forecast earlier than \(t^*_i\), and should
consider forecasting even earlier. This example illustrates the kind of strategic interaction that
the analysts have to take into account. In this section, I develop and prove the existence and
uniqueness of a Subgame Perfect Equilibrium in pure strategies of the timing game between
two analysts.

In the game with two analysts, the second forecaster incorporates the information from
the first forecast. The higher the precision of the investors’ (and the other analyst’s) beliefs
regarding the bias of an analyst, the more the investors can infer from his forecast, and the
higher is the precision of their beliefs immediately after his forecast. Hence, the magnitude of the
discrete jump in the precision of the investors’ beliefs following an analyst’s forecast increases
in both the precision of the analyst’s private signal and in the precision of the investors’ beliefs
about his bias. While forming his strategy, an analyst has to take into account the increase
in the precision of investors’ beliefs due to the other analyst’s forecast, and due to his own
forecast. In contrast to the single analyst case, the precision of the investors’ beliefs regarding
the bias of the analysts does influence their equilibrium timing.

For simplicity, I first solve the model for the particular case where the bias of each analyst
is common knowledge, and hence an analyst’s forecast fully reveals his private signal.\footnote{As will be shown ahead, the equilibrium forecast is linear in the private signal, and hence the forecast fully reveals the private signal.} After
this basic model is established, the case of asymmetric information regarding the analyst’s bias
is introduced.

The basic setup and assumptions are similar to the single analyst case.

4.1 The Known Bias Case

Let’s assume that there are two analysts $i = 1, 2$. The expected utility of analyst $i$ who makes
a forecast at a time $t$ (at which the precision of the investors’ beliefs is $f(t)$) is assumed to be
(as before):

$$EU^i_t = \alpha_i \left( \pi^F_{i,t} - E[\pi | \psi_i, I_i] \right) - \beta_i E \left[ \left( \pi^F_{i,t} - \pi \right)^2 | \psi_i, I_i \right] - f(t),$$

where the parameters of the utility function (including $\alpha_i$) and the precision of the private
signals of the analysts are common knowledge.
Following the arrival of exogenous public information, the precision of the investors’ beliefs is continuously increasing in time, except at the times of the analysts’ forecasts, where it follows a discrete jump. At the beginning of the game (or at any time before the analyst’s forecast), each analyst observes a private signal of the firm’s earnings: \( \tilde{\psi}_i = \pi + \bar{\varepsilon}_i \).

Each analyst has to forecast at some point during the forecasting season \( t \in [0, T] \). An assumption regarding the possibility that both analysts forecast simultaneously at a given point of time is in place. I assume that analysts can forecast simultaneously at \( t = 0 \) or at \( t = T \), in which case the utility of each of them is exactly the same as if he were the only analyst to forecast at that point (similar to the single analyst case). For simplicity of disposition, I assume that for \( t \in (0, T) \) there can be two consecutive forecasts at the same instant of time, so there is no simultaneous forecasting. This assumption holds by applying the framework of Simon and Stinchcombe (1989) (see the relevant footnote in Proposition 2).

A strategy for an analyst is a function that maps from the prior parameters and the analyst’s information into a precision of investors’ beliefs at which to forecast, and the forecast itself at that time. The prior parameters include: the utility function of each analyst, the precision of the analysts’ private signals, and the precision of the investors’ beliefs about the bias parameters of the analysts. I denote the precision of the investors’ beliefs at the equilibrium forecasting time of analyst \( i \) by \( f \left( t^*_{i,c} \right) \) (where the subscript \( c \) indicates the constraint on the precision of the investors’ beliefs at which an analyst can forecast – due to the discrete jump in the precision of the investors’ beliefs following the other analyst’s forecast).

If the increase in the precision of investors’ beliefs following the analysts’ forecasts is sufficiently small, and the unconstrained optimal forecasting times of the analysts are sufficiently apart from each other, then it is feasible that each analyst forecasts at his unconstrained optimal forecasting time – \( t^*_i \) (the timing of the single analyst case). In this case, none of the analysts has an incentive to deviate from this strategy, hence it is an equilibrium. Moreover, I later show that in this case it is the unique Subgame Perfect Equilibrium in pure strategies. But if it is not feasible, then each analyst has to take into account the increase in the precision

---

17 This assumption will be needed to show the uniqueness of the Subgame Perfect Equilibrium. In the case that there is some uncertainty regarding the parameters or regarding the equilibrium timing, or, if we use a stronger concept of equilibrium like \( \varepsilon - \text{equilibrium} \) (or trembling hand perfection) the uniqueness holds even without the above assumption.
of investors’ beliefs due to his own forecast and the other analyst’s forecast. The following Lemma describes the increase in the precision of investors’ beliefs due to an analyst’s forecast.

**Lemma 1** When $\alpha_i$ is common knowledge, the increase in the precision of the investors’ beliefs due to an analyst’s forecast is constant (i.e., independent of the precision of the investors’ beliefs at the time of forecast), and is equal to the precision of the analyst’s private signal.

For proof see Appendix 2.A.

If the precision of investors’ beliefs immediately before the analyst’s forecast is $f(t)$, and the precision of the private signal of analyst $i$ is $f_{S_i}$, then the precision of the investors’ beliefs immediately after the forecast of analyst $i$ is $f(t) + f_{S_i}$.

I next derive the pure strategies Subgame Perfect Equilibrium. I start by deriving the conditions for the corner solutions, then I introduce a definition that will be used in Proposition (2) that provides the full specification of the equilibrium.

**Claim 1** For each analyst $i = 1, 2$:

(A) If $f_{S_i} > \sqrt{\beta_i} - f(0)$ then he forecasts as soon as he can; that is at $t = 0$.

(B) If $f_{S_i} < \sqrt{\beta_i} - f(T)$ then the analyst forecasts at the latest possible time; that is at $t = T$.

**Proof.** This is the unconstrained optimal strategy of an analyst, and it is feasible for both analysts. By revealed preferences this is the optimal strategy.

Let’s consider the case where the unconstrained optimum of both analysts is interior, that is for $i = 1, 2$, $f(0) < f(t^*_i) < f(T)$. Given that the increase in the precision of investors’ beliefs due to the forecast of analyst 2 is $f_{S_2}$, there is a “hole” of size $f_{S_2}$ in the support of the precision of investors’ beliefs at which analyst 1 can publish his forecast. But the location of this “hole” depends on the timing strategy of analyst 2, which of course, takes into account the strategy of analyst 1. To resolve this strategic interaction I will define and use the "indifference interval". Intuitively speaking, the indifference interval of analyst 1 is the interval of precision of investors’ beliefs, of size $f_{S_2}$, for which analyst 1 is indifferent between forecasting at either the lower or the upper end of this interval. Note that in the interior unconstrained optimum case, the expected utility of an analyst monotonically increases in the precision of the investors’ beliefs until it gets maximized, and from there on the expected utility monotonically decreases. This implies
both the uniqueness of the indifferences interval and that the indifferences interval straddles the unconstrained optimum of the analyst (where its expected utility is maximized). Below is a formal definition of the lower end of the indifferences interval ($f_L^1$), which also defines the indifferences interval for more complex cases where the above "intuitively speaking definition" does not exist.

**Definition 2** Define $f_L^1$ as follows:

If there exists a precision of investors’ beliefs $f'$ such that analyst 1 is indifferent between forecasting at a precision of investors’ beliefs that equals either $f'$ or that equals $f' + f_{S_2}$ then $f_L^1 \equiv f'$.\(^{18}\) Figure 2.1 illustrates $f_L^1$ for this case (interior indifferences interval).

If an indifferences interval as the above does not exist, then it is one of the following two cases:

**Case A** - $EU^1 (f(t) = f(0)) > EU^1 (f(t) = f(0) + f_{S_2})$.\(^{19}\) In this case I define $f_L^1 = f(0)$. (See Figure 2.2)

**Case B** - $EU^1 (f(t) = f(T)) > EU^1 (f(t) = f(T) - f_{S_2})$.\(^{20}\) In this case I define $f_L^1$ to be the precision of investors’ beliefs where $EU^1 (f_L^1) = EU^1 (f(T))$ and $f_L^1 < f(T)$. (See Figure 2.3)

\(^{17}\)Up to this point, for the simplicity of disposition, I have not defined whether $f(T)$ is the precision of investors’ beliefs at the end of the “forecasting season” given that the other analyst has or has not published his forecast. Here I can no longer be vague about it, and I define $f(T)$ as the precision of investors’ beliefs given that the other analyst has published his forecast.

---

\(^{18}\)If such $f'$ exists then it is unique and $f(0) \leq f_L^1 < f(t_1)$.

\(^{19}\)Where $EU^1 (f(t) = f(0))$ is the expected utility of analyst 1 given that he forecasts at the time where $f(t) = f(0)$.

\(^{20}\)
Intuitively, $f^1_L$ is the lower end of the interval of precision of investors’ beliefs of size $f_{s_2}$, that will make the analyst indifferent whether he forecasts at the one or the other end of that interval.\footnote{In Case B the interval is smaller than $f_{s_2}$.}

Given the above definition, I can now present the main proposition of this section.

**Proposition 2** There exists a unique Subgame Perfect Equilibrium in pure strategies. The equilibrium strategies of the analysts are as follows:

For each analyst $i = 1, 2$ if $f^1_L = f(0)$ he forecasts at $t = 0$. If for at least one of the analysts $f^1_L > f(0)$, then analyst 1 is the first to forecast if and only if $f^1_L < f^2_L$.\footnote{I could use a different tie breaking rule where analyst 1 is the first to forecast iff $f^1_L \leq f^2_L$.} Let’s assume w.l.o.g. that analyst 1 is the first to forecast. Analyst 1 forecasts at a time $t^*_1,c$ where the precision of the investors’ beliefs is:

$$f(t^*_1,c) = \min \{ f^2_L, f(t^*_1) \}.$$  

If analyst 1 forecasts at $f^2_L$ then the analyst 2 forecasts “immediately after”, where the precision of the investors’ beliefs is $f(t) = f^2_L + f_{s_1}$.\footnote{The strategy which predicts that the second analyst will forecast “immediately after” the first analyst is somewhat vague. One way to have well defined strategies and outcomes is using the framework of Simon and Stinchcombe (1989). All three assumptions that they impose on the strategies (F1-F3) hold in my model. Using this framework, there can be two consecutive forecasts at the same instant of time (they show that the limit of the discrete time equilibria as the time interval goes to zero converges to the continuous time equilibrium). Another way to have well defined strategies is using the framework of Perry and Reny (1994), where restriction (S4) upon strategies imposes some $\varepsilon$ “lag” between agents’ actions and guarantees that the game is well defined. Adopting the framework of Perry and Reny requires some adjustments for $f^1_L$ (in a magnitude smaller than $\varepsilon$).} If the first analyst forecasts at $f(t^*_1)$ then the second analyst will forecast at $f(t^*_2)$ if it is feasible (that is if: $f(t^*_1) + f_{s_1} \leq f(t^*_2)$), or else...
“immediately after” the first forecast.

The optimal forecast of every analyst $i$ is:

$$\pi_{i,t}^{F} = \frac{\alpha_i}{2\beta_i} + E(\pi_i|\psi_i, I_t).$$

The off equilibrium beliefs are as follows. Let’s assume w.l.o.g. that $f_{L1}^1 < f_{L2}^2$. For all $f(t) > f_{L2}^2$, if no forecast was published analyst 1 believes that analyst 2 is going to forecast immediately.\(^{24}\)

Before proving the proposition, I describe the equilibrium intuitively and graphically.

The equilibrium may have one of the following two patterns. Non Clustering Equilibrium Pattern (separation in time) – each analyst publishes his forecast at his unconstrained optimum.\(^{25}\) If this is not feasible, then the equilibrium is of the second pattern. Clustering in Time Pattern – the first forecaster is the analyst who’s lower end of the indifference interval ($f_{L1}^i$) is smaller. Assume this is analyst 1. Analyst 1 publishes his forecast at $f_{L2}^2$. Following his forecast, the precision of the investors’ beliefs increases instantaneously and becomes higher than the unconstrained optimum of analyst 2. At that point, the expected utility of analyst 2 decreases in the precision of the investors’ beliefs, and hence he publishes his forecast immediately. Both patterns are presented in the following two figures.

---

\(^{24}\)The equilibrium can be supported by a larger and more general set of off-equilibrium beliefs, including mixed strategies off the equilibrium path.

\(^{25}\)The Non-Clustering Pattern may be feasible even in the case where the unconstrained optimum of an analyst is included in the indifference interval of the other analyst.
Since $f_{1L} < f_{2L}$ analyst 1 is the first to forecast. He publishes his forecast at $f_{1L}$. Following his forecast, the precision of the investors beliefs “jumps” to $f_{1L} + f_{S2}$ which is higher than the unconstrained optimum of analyst 2. Since the expected utility of analyst 2 at this region is decreasing in the precision of the investors’ beliefs, analyst 2 publishes his forecast immediately after the forecast of analyst 1.

Proof of Proposition 2. Analyst 1 will never forecast earlier than $f_{1L}$ since he can guarantee himself an expected utility of at least $EU^1(f(t) = f_{1L}) = EU^1(f(t) = f_{1L} + f_{S2})$. If the precision of investors’ beliefs is higher than $f(t^*)$ (due to a discrete jump after the forecast of analyst 2), then analyst 1 forecasts immediately. Hence, the only interval of precision of investors’ beliefs left to investigate (from analyst 1’s perspective) is $f_{1L} < f(t) < f(t^*)$. In the case where $f_{2L} > f(t^*)$ analyst 2 will not forecast before $f_{2L}$, and analyst 1 will wait and forecast at his unconstrained optimum (where the precision is $f(t^*)$). In the case where $f_{1L} < f_{2L} < f(t^*)$ it is straightforward that analyst 1 will wait at least until $f_{2L}$.

Before proceeding with the proof of the Proposition, I introduce the following Lemma.

Lemma 2 If $f_{1L} < f_{2L} < f(t^*)$ then there is no pure strategies Subgame Perfect Equilibrium in which the first forecast of the analysts will be at $f(t) > f_{2L}$.

Proof of the Lemma. Assume that such pure strategies Subgame Perfect Equilibrium exists. Then, the second forecaster can deviate and forecast at a sufficiently small amount of time earlier than the first forecaster, and by doing so he strictly increases his expected utility – in contradiction to this being an equilibrium. QED Lemma.

The Lemma indicates that for $f_{1L} < f_{2L} < f(t^*)$, if analyst 2 has not forecasted before $f_{2L}$, analyst 1 will forecast at $f_{2L}$. 

20
So far, I have shown that if \( f^1_L < f^2_L \) then analyst 1 will forecast first at a precision of investors’ beliefs of \( \min \{ f^2_L, f(t^*_1) \} \). It is straightforward that the second analyst to forecast will either wait until \( f(t^*_2) \) (if it is feasible, that is if: \( f(t^*_2) > f^2_L + f_{S_1} \)), or else he will forecast immediately after the first analyst. QED.

4.1.1 Discussion of the Known Bias Case

The central questions of this paper are: what determines the order and timing of the analysts’ forecasts, and what are the empirical predictions generated by the model? In the model, the order and timing of the forecasts are determined by the following factors: the precision of the private signals of the analysts \( f_{S_i} \), the reputation parameters of the analysts \( \beta_i \), and the process of exogenous information arrival (in the Unknown Bias case that will be presented in the following section, also the precision of the investors’ beliefs about the bias parameter \( \alpha_i \), influences the order and timing of the forecasts). I next elaborate on the influence of each of the above factors on the equilibrium order and timing of the analysts’ earnings forecasts.

An increase in the reputation cost parameter of analyst \( i (\beta_i) \) motivates him to publish his forecast later – when there is more public information that he can use. More formally, it pushes the analyst’s unconstrained optimum to later in time where the precision of the investors’ beliefs is higher. Note that the size of the indifference interval of both analysts is independent of \( \beta_i \). Hence, an increase in \( \beta_i \) shifts the indifference interval of analyst \( i \) to the right, without affecting the indifference interval of the second analyst. This will induce a later forecast by analyst \( i \) (except for the clustering pattern case where analyst \( i \) forecasts immediately after the other analyst, and still does so after the increase in \( \beta_i \)). Note that if analyst \( i \) was the first to forecast, the increase in \( \beta_i \) may also change the order of the forecasts.

Changes in the process of exogenous information arrival will not influence the precision of the investors’ beliefs at which each of the analysts will publish his forecast. That is, with respect to the "precision of the investors’ beliefs time line" nothing changes. The only thing that does change is the calendar time at which each analyst publishes his forecast.

An increase in the precision of the private signal of analyst \( i \) has two conflicting effects on the equilibrium order of forecasts. On the one hand, the unconstrained optimum of analyst \( i \) is now at a lower precision of investors’ beliefs. On the other hand, the discrete jump in the precision
of investors’ beliefs following the forecast of analyst $i$ ($\Delta f_{S_i} = f_{S_i}$) becomes bigger, which increases the indifference interval of analyst $j$, and reduces the lower end of the indifference interval of analyst $j - f^j_L$. This in turn (all else equal), induces earlier forecast by analyst $j$ (in the clustering pattern equilibrium). The corollary bellow indicates that the influence of the first effect on the order of forecasts always dominates.

**Corollary 2 (Comparative Statics)** Suppose that in equilibrium, analyst $i$ forecasts at time $t^* \in (0,T)$. An increase in the precision of the private signal of analyst $i$ will advance the time of his forecast, and weakly advance the forecasting time of analyst $j$. If analyst $i$ was the first forecaster, he will still forecast first, but if he was the second forecaster, then he may now become the first to forecast. Moreover, the precision of the investors’ beliefs immediately prior to the first forecast will be lower relative to the time before the change.$^{26}$

For the proof of the Corollary see Appendix 3.

An interesting feature of the equilibrium is that it is not necessarily the analyst with the higher precision of private signal who will be the first to forecast. The unconstrained optimum of an analyst is determined by the combination of the precision of his private signal ($f_{S_i}$) and the error/reputation cost parameter ($\beta_i$). It is possible that analyst $i$ has a higher precision of private signal, but his error cost parameter is sufficiently higher than that of analyst $j$, so that the unconstrained optimum of analyst $i$ will be at a higher precision of the investors’ beliefs. This indicates that when comparing two different single analyst cases, it might be that the unconstrained optimum of an analyst with higher precision of private signal is at a later time than of the other analyst. But in the forecasting timing game, it is also possible that even though the unconstrained optimum of analyst $i$ is at a lower precision of investors’ beliefs than the unconstrained optimum of analyst $j$, still analyst $j$ will be the first to forecast. This can happen because the higher precision of the private signal of analyst $i$ induces a bigger indifference interval of analyst $j$, which causes $f^j_L$ to be lower than $f^i_L$. Since the first analyst to forecast is the one who demonstrates a smaller lower end of the indifference interval ($f^k_L$), it might be that $f^j_L < f^i_L$. If this is the case, then analyst $j$ will be the first to forecast.

---

$^{26}$The precision of investors’ beliefs at the time of the second forecaster may be higher, the same or lower. Both cases are presented in the proof.
4.1.2 A Note on Multi-Analysts’ Timing Game

A very interesting extension of the model is the multi-analysts’ timing game, where there are more than two analysts. Having more than two analysts complicates the model. In a multi-analysts’ game, the “indifference interval” of each analyst should be defined in respect to each of the other analysts, since the increase in the precision of the investors’ beliefs following the forecasts of different analysts may be different. In this case, it is not clear whether the uniqueness of the equilibrium holds in the general case, and I suspect that there is no closed form solution for deriving the equilibrium. Nevertheless, I conjecture that the intuitions of the two-analysts’ game apply also to the multi-analysts’ game. If it is feasible that each analyst will publish his forecast at his constrained optimum then it is obviously a Subgame Perfect Equilibrium – the Non Clustering Pattern. Intuitively, one can think about this case as "no overlap" – in the sense that none of the analysts’ unconstrained optimum is included in an other analyst’s indifference interval.\textsuperscript{27} If there exists an "overlap" between at least two analysts, then in equilibrium we may obtain clustering in time of the analysts involved in this "overlap". Intuitively speaking, as long as the indifference interval of each analyst includes the unconstrained optimum of at most one other analyst, the model can be implemented for any number of analysts. In this case we will obtain a combination of clusters of at most two forecasts, as well as separation in time between different clusters and between single forecasts that are not clustered. This may occur if the unconstrained optimum of the analysts is not “too close” to each other, and the increase in the precision of the investors’ beliefs due to the analysts’ forecasts is not “too big”. Roughly speaking, the more analysts there are, the more likely there will be "overlap", hence the more likely that we will obtain clustering in time.

Another prediction from increasing the number of analysts is that the analysts will forecast weakly earlier in time, which was obtained also while adding a second analyst to the single analyst case.

\textsuperscript{27}For brevity, the term "overlap" is not well defined. In order for it to be well defined all the permutations of orders should be considered.
4.2 The Unknown Bias Case

I believe that in “real life”, the investors do not know the exact incentives of an analyst (even if he is an affiliated analyst) at a given point in time to bias his forecast. In this section, I assume that the realization of the analyst’s bias parameter ($\alpha_i$) is not known to the investors, i.e., it is the analyst’s private information. An analyst has two private signals (his bias parameter ($\alpha_i$) and the signal about the firm’s earnings ($\psi_i$)), and he provides only one signal to the investors – his forecast. As a result (contrary to the known bias case), the forecasts of the analysts do not fully reveal their private signals. But the main difference between the Known and Unknown bias cases is more than just the additional noise in the signal obtained by the investors. The known bias case was a static game, in the sense that the increase in the precision of the investors’ beliefs following an analyst’s forecast was independent of the time of the forecast. This means that the indifference intervals of the analysts are constant over time, and the only thing that changes over time is the precision of the investors’ beliefs. As will be shown below, this is not the case in the unknown bias case. The unknown bias case is dynamic in the sense that the discrete increase in the precision of the investors’ beliefs following an analyst’s forecast is not constant, and depends on the precision of the investors’ beliefs immediately prior to the forecast. This means that the indifference intervals of the analysts change over time. The strategic decision of the analysts becomes more complicated since they have to consider the changes over time in their own and in the other analyst’s indifference intervals.

I start with deriving and analyzing the increase in the precision of investors’ beliefs following an analyst’s forecast, and then I present and discuss the equilibrium.

Appendix 2.B shows that immediately after the forecast of analyst $i$ (at time $t$), the variance of the investors’ beliefs about the earnings is given by:

$$\operatorname{Var} (\pi_t | \pi_{i,t}^F, I_t) \equiv \operatorname{Var} (\pi_t | \pi_{i,t}^F) = \sigma_{\pi_t}^2 \left( 1 - \rho_{\pi_i,\pi_t}^2 \right)$$

$$= \sigma_{\pi_t}^2 \left( 1 - \frac{\frac{\sigma_{\pi_t}^2}{\sigma_{\pi_i}^2 + \sigma_i^2} \sigma_{\pi_t}^2 \left( \frac{1}{(2\pi_i)^2} \sigma_{\pi_i}^2 + \frac{\sigma_i^2}{\sigma_{\pi_i}^2 + \sigma_i^2} \sigma_{\pi_t}^2 \right)}{\frac{\sigma_{\pi_t}^2}{\sigma_{\pi_i}^2 + \sigma_i^2} \sigma_{\pi_t}^2 + \sigma_{\pi_t}^2 + \sigma_i^2} \right)$$

(4)

where $\sigma_{\pi_t}^2 \equiv \operatorname{Var} (\pi_t | I_t) \equiv \frac{1}{f(t)}$ is the variance of the investors’ beliefs about the firm’s earnings at time $t$ – before the analyst’s forecast, $\rho_{\pi_i,\pi_t}^2$ is the correlation coefficient between the analyst’s
forecast and the firm’s earnings, and \( \sigma_{\alpha_i}^2 \) is the variance of the investors’ beliefs about the analyst’s bias parameter - \( \alpha_i \).

Denoting the increase in the precision of the investors’ beliefs following a forecast of analyst \( i \) at time \( t \) by \( \Delta f_{S_i}(t) \), we have:

\[
\Delta f_{S_i}(t) \equiv \frac{1}{\text{Var}(\pi_i|\pi_{i,t}^F)} - \frac{1}{\text{Var}(\pi_i|I_t)},
\]

Note that \( \Delta f_{S_i}(t) \) is not constant and depends on \( t \) (or on \( f(t) \)). In contrast to the known bias case where the “indifference interval” (and hence \( f_L^i \)) of each analyst was constant over time, here it is not the case. Finding an equilibrium in such case may seem complicated, and one can predict multiple equilibria. Fortunately this is not the case and the equilibrium is unique.

It can be easily obtained from equation (4) that lower \( \sigma_{\alpha_i}^2 \) and higher precision of the analyst’s private signal (\( f_{S_i} \equiv \frac{1}{\sigma_{\alpha_i}^2} \)) increase the amount of information conveyed by the analyst’s forecast, and hence the precision of the investors’ beliefs following his forecast.

The following Lemma presents a result which is not straightforward. The Lemma will be used in deriving the equilibrium and proving its uniqueness.

**Lemma 3**

\[
0 > \frac{\partial \Delta f_{S_i}(t)}{\partial f(t)} > -1.
\]

**Proof:** See Appendix 2.C.

The Lemma says that \( \Delta f_{S_i}(t) \) monotonically decreases in the precision of investors’ beliefs immediately prior to the analyst’s forecast. Moreover, the decrease in \( \Delta f_{S_i}(t) \) is lower than the increase in \( f(t) \).

As time progresses, the precision of the investors’ beliefs increases and \( \Delta f_{S_i}(t) \) decreases. It means that the indifference interval of analyst \( j \) decreases (note that the indifference interval of analyst \( j \) is of size equal to \( \Delta f_{S_i}(t) \) and it straddles the unconstrained optimum of analyst \( j \)). Since \( \Delta f_{S_i}(t) \) depends on \( t \), the lower end of the indifference interval of analyst \( j \) (which was denoted before as \( f_L^j \)) also depends on \( t \). I add to its notation the subscript \( t \), and it is now denoted by: \( f_L^j(t) \). If the decrease in the size of the indifference interval was sufficiently fast, then we could expect a situation in which as \( f(t) \) increases \( f_L^j(t) \) increases even faster. This
in turn could induce that \( f(t) \) will equal \( f_L^j(t) \) more than once during the forecasting season, which would at least distort the uniqueness of an equilibrium similar to the known bias case. Lemma 3 implies that after the first time where \( f(t) = f_L^j(t) \), we get that \( f(t) \) will always be higher than \( f_L^j(t) \). This proves the uniqueness of the time at which \( f(t) = f_L^j(t) \).

The main proposition of the model uses the following notation.

**Notation 3** \( \tau_0 \) Denotes the first time where both \( f(t) \geq f_L^1(t) \) and \( f(t) \geq f_L^2(t) \).

**Proposition 3** There exists a unique Subgame Perfect Equilibrium in pure strategies where:

Analyst 1 is the first to forecast if and only if \( f_1^1(\tau_0) < f_2^2(\tau_0) \).

The precision of the investors’ beliefs immediately prior to the forecast of the first analyst (assume w.l.o.g. it is analyst 1) is

\[
 f(t^*_1) = \min \{ f_L^2(\tau_0), f(t^*_1) \}. \tag{28}
\]

Analyst 2 will forecast at his unconstrained optimum if it is feasible, otherwise he will forecast immediately after the forecast of analyst 1.\(^29\)

The forecasts of the analysts are:

\[
 \text{for } k = 1, 2: \quad \pi_k = \frac{\alpha_k}{2\beta_k} + E(\pi_k, I_t)
\]

**Proof.** As in the known bias case, it is straightforward that as long as for at least one of the analysts \( f_L^k(t) > f(t) \), none of the analysts publishes his forecast. As in the known bias case, if it is feasible that each analyst will publish his forecast at his unconstrained optimum - \( f(t^*_k) \), doing so is an equilibrium. If it is not feasible, the first candidate for a pure strategies Subgame Perfect Equilibrium is the following: at \( \tau_0 \), the analyst with the lower \( f_L^1(\tau_0) \) (w.l.o.g. analyst 1) will forecast first. If analyst 1 did not forecast at \( f_L^2(\tau_0) \) then analyst 2 will forecast immediately after (this is off the equilibrium path). Since non of the analysts will want to deviate, it is a subgame perfect equilibrium.

I next prove the uniqueness of this pure strategies equilibrium. Let us assume that there is

\(^{28}\)The first forecaster does not necessarily publish his forecast at \( t = \tau_0 \). This is the case only in the clustering pattern of the equilibrium. In the separation in time pattern, the first forecaster will forecast earlier than \( \tau_0 \).

\(^{29}\)The notes about ”immediately after” and about the off equilibrium beliefs in Proposition 2 (the Known Bias Case) apply here as well.
another pure strategies Subgame Perfect Equilibrium, in which the first analyst to forecast publishes his forecast at a time later than \( \tau_0 \). In this case, the expected utility of the other analyst (the second forecaster) is lower than if he himself was forecasting at \( \tau_0 \) (or a sufficiently small amount of time before that), hence he will want to deviate – thus contradicting the assumption that this is an equilibrium. Noting that each analyst will forecast immediately as he can whenever the precision of the investors’ beliefs is higher than his unconstrained optimum, completes the proof for the existence and uniqueness of the above equilibrium. □

4.2.1 Discussion of the Unknown Bias Case

Introducing a noise in the investors’ (and the other analyst’s) beliefs about the bias of the analysts adds another layer of complexity to the model. From the investors’ (and the other analyst’s) perspective, the signal about the earnings of the firm that they infer from the analyst’s forecast is noisier relative to the known bias case. The higher the noise in the investors’ beliefs about the analyst’s bias, the noisier is the signal they infer from the analyst’s forecast. In addition to direct informational effect, the precision of the beliefs about the analysts’ biases plays a role in the equilibrium timing of the analysts as well. The higher the precision of the investors’ beliefs about an analyst’s bias, the more information the investors (and the other analyst) infer from the forecast, and hence the larger is the indifference interval of the other analyst. This influences the equilibrium forecasting timings of the analysts.

All the points raised in the discussion of the known bias case hold in the unknown bias case as well. The only point that is not straightforward and should be proved is Corollary 2. In order to show that Corollary 2 of the known bias case holds in the unknown bias case as well, it is sufficient to show that: \( 0 < \frac{\partial \Delta f_{S_i}(t)}{\partial f_{S_i}} < 1 \). This is proved in Appendix 2.D. As \( f_{S_i} \) increases we get that \( \Delta f_{S_i}(t) \) increases as well, but by less than the increase in \( f_{S_i} \). It means that the indifference interval of analyst \( j \) increases in \( f_{S_i} \), but by less than the increase in \( f_{S_i} \). Other than this qualitative change in respect to the known bias case, the rest of the proof of Corollary 2 remains the same. As to the quantitative effect, in the unknown bias case the forecasts of the first analyst will be earlier as \( f_{S_i} \) increases, but by a smaller extent than in the known bias case.

As in the known bias case, the analyst with the higher precision of private signal will not
necessarily be the first to forecast. In the unknown bias case, there is an additional "degree of freedom" that can influence the order of the forecasts. Due to the additional layer of asymmetric information, if the precision of the investors’ beliefs about the bias of analyst $i$ is higher than about the bias of analyst $j$, then even if the unconstrained optimum of analyst $i$ is higher, and the analysts have an equal precision of private signals, it is still possible that analyst $i$ will be the second to forecast.

The equilibrium forecasting timing is invariant to the realizations of the private signals and the realized bias parameters of the analysts. Given estimates of the analysts’ parameters and the precision of the investors’ beliefs, the model generates precise predictions of the analysts’ forecasting timing. The model does not impose any restrictions on the exogenous stream of information. In the data, we often see that analysts’ forecasts are clustered around an “informational event” related to the firm. If during a given period there is an extensively incoming flow of information, we get that the growing frequency of forecasts around this period arises endogenously from the model.\footnote{In case of a discrete increase in the public information, it can be shown that the analysts are likely to cluster and forecast simultaneously.}

\section{Conclusions}

The paper proposes a theoretical model for the order and timing of analysts’ earnings forecasts. The question that generated the paper is whether the order and timing of analysts' earnings forecasts are determined exogenously, as implicitly assumed in much of the literature, or whether analysts choose the timing of their forecasts strategically. The answer to this question may yield important insights into the behavior of sell-side analysts, a subject that has received much recent attention. Understanding the information contained in the order of analysts’ forecasts may help investors and academics alike to decipher the informational content of these forecasts.\footnote{In a Guttman (2005), I analyze the incorporation of the timing of the analysts’ forecasts in deriving consistent investors’ beliefs following every analyst’s forecast.} I follow the literature in assuming that analysts care foremost about the accuracy of their forecasts, but in some cases they may have an incentive to bias their forecasts. I also add an additional feature by assuming that the compensation of the analyst declines in the precision of the
investors’ beliefs about earnings at the time of the forecast. Thus, an early-reporting analyst is compensated more for a given forecast than a later-reporting one. I further assume that a continuous stream of information from sources other than analysts’ forecasts arrives during the quarter and affects the investors’ beliefs about the firm’s earnings. The analysts face the trade-off between an earlier, but less precise forecast, versus a later, but more precise forecast. Since the investors do not necessarily learn the actual bias of the analysts, I assume that they only know the distribution from which the analysts’ bias is drawn.

The paper introduces a strategic game between two analysts, and derives the unique pure strategies Subgame Perfect Equilibrium. It is shown that: More precise private signal, less precise investors’ beliefs regarding the analysts’ bias, and lower error/reputation costs will all induce an earlier forecast. The equilibrium has two patterns: Either each analyst forecasts at his unconstrained optimal time (as if he is the only analyst that covers the firm), or the forecasts cluster in time, with one analyst forecasting immediately after the other. An extensive inflow of information (e.g., around an event) endogenously generates more forecasts around that period.
References


6 Appendix

Appendix 1. The Single Analyst’s Case - a More General Case

This appendix solves the Single Analyst’s case, assuming a more general assumption regarding the functional relation between the analyst’s expected payoff and the precision of the investors’ beliefs at the time of the forecast.

Assuming that the expected utility of analyst $i$ is:

$$EU_t^i = \alpha_i \left( \pi_{i,t}^F - E[\pi|\psi_i, I_t] \right) - \beta_i E \left[ (\pi_{i,t}^F - \pi)^2 |\psi_i, I_t \right] - \gamma_i g (f(t))$$

where $\gamma_i$ is a positive constant and $g(\cdot)$ is a continuously increasing function.

**Optimal Forecast**

The first order condition with respect to the forecast (at time $t$) is

$$\frac{\partial U_t^i}{\partial \pi_t^F} = \alpha_i - 2\beta_i E \left( (\pi_{i,t}^F - \pi) |\psi_i, I_t \right) = 0.$$

The second order condition is $\frac{\partial^2 U_t^i}{\partial (\pi_t^F)^2} = -2\beta_i < 0$. Hence, for every given timing $t$, in which the analyst chooses to report, his optimal report will be:

$$\pi_{i,t}^F = \frac{\alpha_i}{2\beta_i} + E(\pi|\psi_i)$$

$$= \frac{\alpha_i}{2\beta_i} + \mu_{\pi_t} + \frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\psi_i}^2} (\psi_i - \mu_{\psi})$$

where $\mu_{\pi_t}$ is the public expectation at time $t$ (which may be different than $\mu_{\pi_0}$), and $\mu_{\psi}$ is the expected private signal, given all the public information at time $t$.

Substituting the forecast of the analyst into his utility function yields:

$$EU_t^i = E \left[ \left( \alpha_i (\pi_{i,t}^F - \pi) - \beta_i \left( \frac{\alpha_i}{2\beta_i} + E(\pi|\psi_i) - \pi \right)^2 - \gamma_i g (f(t)) \right) |\psi_i, I_t \right]$$

$$= \frac{\alpha_i^2}{2\beta_i} - \beta_i \left( \frac{\alpha_i}{2\beta_i} \right)^2 - \beta_i E \left[ (E(\pi|\psi_i) - \pi)^2 |\psi_i, I_t \right] - 0 - \gamma_i g (f(t))$$

$$= \frac{\alpha_i^2}{4\beta_i} - \frac{\beta_i}{f(t) + f_S} - \gamma_i g (f(t))$$

**Optimal Forecasting Time**
In order to find the optimal forecasting timing, I derive the analyst’s expected utility with respect to the precision of the investors’ beliefs.

The FOC with respect to the optimal forecasting timing is:

\[
\frac{\partial EU_i}{\partial f(t)} = \frac{\partial}{\partial f(t)} \left[ \frac{\alpha_i^2}{4\beta_i} - \frac{\beta_i}{f(t) + f_S} - \gamma_i g(f(t)) \right] = \frac{\beta_i}{(f(t) + f_S)^2} - \gamma_i \frac{\partial g(f(t))}{\partial f(t)} = 0
\]

The SOC is:

\[
\frac{\partial^2 EU_i}{\partial f(t)^2} = \frac{\partial}{\partial f(t)} \left[ \frac{\beta_i}{(f(t) + f_S)^2} - \gamma_i \frac{\partial g(f(t))}{\partial f(t)} \right] = -\frac{\beta_i}{(f(t) + f_S)^3} - \gamma_i \frac{\partial^2 g(f(t))}{\partial f(t)^2}
\]

A necessary condition for interior utility maximizer is that for the timing where the FOC holds, the following holds: \(\frac{\partial^2 g(f(t))}{\partial f(t)^2} \geq -\frac{\beta}{\gamma(f(t) + f_S)^3}\).

**Corollary 3** If \(f_S > \sqrt[3]{\frac{\beta_i}{\gamma_i \partial g(f(t)) / \partial f(t)}}\) then the expected utility of the analyst decreases in time, and he will forecast at \(t = 0\).

For \(f_S < \sqrt[3]{\frac{\beta_i}{\gamma_i \partial g(f(t)) / \partial f(t)}}\), the unconstrained optimal forecasting timing of the analyst is

\[
f(t)^* = \sqrt{\frac{\beta_i}{\gamma_i \partial g(f(t)) / \partial f(t)}} - f_S,
\]

imposing the constraint of the precision of the investors’ beliefs in the forecasting season, we get:

If \(f(t)^* \leq f(0)\) then the analyst will forecast at \(t = 0\).

If \(f(0) \leq f(t)^* \leq f(T)\) then the analyst will forecast at the time at which \(f(t) = f(t)^*\).

If \(f(t)^* > f(T)\) then the analyst will forecast at time \(T\).

---

**Appendix 2 - The Increase in the Precision of the Investors’ Beliefs Following an Analyst’s Forecast**
In this appendix I show the following:

A. Proof of Lemma 1 – In the case where the analyst’s bias is common knowledge, the increase in the precision of the investors’ beliefs due to an analyst’s forecast equals the precision of the analyst’s private signal.

B. In the Unknown analyst’s bias case, I derive the increase in the precision of the investors’ beliefs due to an analyst’s forecast (as a function of the precision of investors’ beliefs prior to the analyst’s forecast, the precision of the investors’ beliefs about the analyst’s bias and the precision of the analyst’s private signal).

C. Proof of Lemma 3 (Unknown Bias case).

D. For the Unknown Bias case – \( 0 < \frac{\partial \Delta f_S(t)}{\Delta f_S} < 1 \). That is, when \( f_S \) increases by a marginal unit \( \Delta f_S \), will increase by less than a marginal unit.

To simplify the notation, I omit the subscript \( i \) to indicate the specific analyst.

2.A. The common knowledge case - Proof of Lemma 1

Let’s assume that the bias parameter \( \alpha_i \) of the analyst is common knowledge, and that the precision of investors’ beliefs before the analyst’s forecast is \( f(t) \). Since the analyst’s forecast fully reveals his private signal, the posterior precision of the investors’ beliefs equals the posterior precision of the analyst’s beliefs, and is: (using the conditional variance formula)

\[
Var(\pi|\pi^F) = Var(\pi|\psi) = \sigma^2_\pi (1 - \rho^2_{\pi|\psi}) = \sigma^2_\pi \left(1 - \left(\frac{\sigma^2_\pi}{\sqrt{\sigma^2_\pi + \sigma^2_\varepsilon}}\right)^2\right) = \sigma^2_\pi \left(1 - \frac{\sigma^2_\pi}{\sigma^2_\pi (\sigma^2_\pi + \sigma^2_\varepsilon)}\right) = \frac{\sigma^2_\pi}{\sigma^2_\pi + \sigma^2_\varepsilon} \sigma^2_\pi
\]

where \( \rho^2_{\pi|\psi} \) is the correlation coefficient.

Hence, the posterior precision denoted by \( f^+(t) \) is

\[
f^+(t) = \frac{1}{Var(\pi|\psi)} = \frac{\sigma^2_\pi + \sigma^2_\varepsilon}{\sigma^2_\pi \sigma^2_\varepsilon} = \frac{1}{f(t)} + \frac{1}{f_S} = \frac{f_S + f(t)}{f(t)f_S} = f_S + f(t)
\]

QED.

2.B. The change in the precision of the Investors' beliefs due to an analyst’s forecast - Unknown Bias Case
By applying the conditional variance formula we get:

\[
\pi_t^F = \frac{\alpha}{2\beta} + E(\pi_t | \psi) = \frac{\alpha}{2\beta} + \mu_{\pi_t} + \frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2}\left(\psi - \mu_\psi\right); \quad \mu_{\pi_t}^F = \frac{\mu_{\alpha_t}}{2\beta} + \mu_{\pi_t}
\]

Proof. Keeping the precision of the analyst’s private signal and the precision of investors’ beliefs about the analyst’s bias constant, it is sufficient to show that

\[
\frac{\partial}{\partial \text{Var}(\pi_t)} \left(\frac{1}{\text{Var}(\pi_t | \pi^F)} - \frac{1}{\text{Var}(\pi_t)}\right) < 0
\]

By applying the conditional variance formula we get:

\[
\text{Var}(\pi_t | \pi_t^F) = \frac{1}{(2\beta)^2} \sigma_\alpha^2 + \left(\frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2}\right)^2 \left(\sigma_{\pi_t}^2 + \sigma_\varepsilon^2\right) = \frac{1}{(2\beta)^2} \sigma_\alpha^2 + \frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2} \sigma_{\pi_t}^2
\]

\[
\text{Cov}((\pi_t, \pi_t^F)) = \frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2}
\]

\[
E(\pi_t | \pi_t^F) = \mu_{\pi_t} + \frac{\text{Cov}(\pi_t, \pi_t^F)}{\text{Var}(\pi_t)}(\pi_t^F - \mu_{\pi_t}) = \mu_{\pi_t} + \frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2} \sigma_{\pi_t}^2 = \mu_{\pi_t} + \frac{\frac{1}{(2\beta)^2} \sigma_\alpha^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2} \sigma_{\pi_t}^2 + \frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2} \frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2} \pi_t^F
\]

\[
\rho_{\pi_t^F, \pi_t}^2 = \frac{\left(\frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2}\right)^2}{\sigma_{\pi_t}^2 \left(\frac{1}{(2\beta)^2} \sigma_\alpha^2 + \frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2} \sigma_{\pi_t}^2\right)} = \frac{\left(\frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2}\right)^2}{\sigma_{\pi_t}^2 \left(\frac{1}{(2\beta)^2} \sigma_\alpha^2 + \frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2} \sigma_{\pi_t}^2\right)}
\]

By applying the conditional variance formula we get:

\[
\text{Var}(\pi_t | \pi_t^F) = \sigma_{\pi_t}^2 \left(1 - \rho_{\pi_t^F, \pi_t}^2\right)
\]

\[
= \sigma_{\pi_t}^2 \left(1 - \frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2} \frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2} \sigma_{\pi_t}^2\right) = \sigma_{\pi_t}^2 \left(1 - \frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2} \frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi}^2} \sigma_{\pi_t}^2\right).
\]

2.C. Proof of Lemma 3.

Claim 4 The increase in the precision of the investors’ beliefs following the analyst’s forecast is decreasing in the precision of the investors’ beliefs prior to the analyst’s forecast.

Proof. Keeping the precision of the analyst’s private signal and the precision of investors’ beliefs about the analyst’s bias constant, it is sufficient to show that

\[
\frac{\partial}{\partial \text{Var}(\pi_t)} \left(\frac{1}{\text{Var}(\pi_t | \pi^F)} - \frac{1}{\text{Var}(\pi_t)}\right) < 0
\]
Rewriting \( \frac{1}{\text{Var}(\pi_t|\pi^F)} - \frac{1}{\text{Var}(\pi_t)} \) yields:

\[
\frac{1}{\text{Var}(\pi_t|\pi^F)} - \frac{1}{\text{Var}(\pi_t)} = \left( \frac{1}{\sigma_{\pi_t}^2} \right) \frac{1}{1 - \frac{\left( \frac{\sigma_{\pi_t}^2}{\sigma_{\pi_t}^2 + \sigma_{\pi^F}^2} \right)^2}{\sigma_{\pi_t}^2 + \sigma_{\pi^F}^2}} - \frac{1}{\sigma_{\pi_t}^2}
\]

Denoting \( \sigma_{\pi_t}^2 \equiv x \), \( \sigma_{\pi^F}^2 \equiv y \), \( \frac{1}{(2\beta)^2} \sigma_{\epsilon_t}^2 \equiv z \), and taking the derivative of the above with respect to \( x \) we get:

\[
\frac{\partial}{\partial x} \left( \frac{1}{x \left( 1 - \frac{\left( \frac{1}{x+y} \right)^2}{z+\left( \frac{1}{x+y} \right)^2} \right)} - \frac{1}{x} \right) = \frac{2zx y}{(zx^2 + 2zxy + y^2 + yx^2)^2} > 0
\]

This means that the increase in the precision of investors’ beliefs due to the analyst’s forecast increases as the prior variance increases. That is, the increase in the precision of investors’ beliefs due to the analyst’s forecast decreases as the precision of investors’ beliefs prior to the analyst’s forecast increases.

QED Claim 4 ■

I now prove the second part of Lemma 3.

Denoting \( b = \frac{1}{x} \), it is left to show the following claim.

\[\text{Claim 5 For all } b > 0 \text{ we have}\]

\[
0 > \frac{\partial}{\partial b} \left( \frac{1}{\frac{1}{b} \left( 1 - \frac{\left( \frac{1}{x+y} \right)^2}{z+\left( \frac{1}{x+y} \right)^2} \right)} - b \right) > -1
\]

\[\text{Proof. Denote: } w = \frac{1}{y} \text{, } v = \frac{1}{z} \text{ and rewriting we get:}\]

\[
\frac{\partial}{\partial b} \left( \frac{1}{\frac{1}{b} \left( 1 - \frac{\left( \frac{1}{x+y} \right)^2}{z+\left( \frac{1}{x+y} \right)^2} \right)} - b \right) > -1
\]
Taking the derivative and collecting terms yields that the following has to be proved:

\[-\frac{2(w + b)w^2v}{(w^2 + 2bw + b^2 + wv)^2} > -1\]

or equivalently:

\[ (w^2 + 2bw + b^2 + wv)^2 - 2(w + b)w^2v > 0 \]

However:

\[ (w^2 + 2bw + b^2 + wv)^2 - 2(w + b)w^2v = ((w + b)^2 + wv)^2 - 2(w + b)w^2v \]
\[ = (w + b)^4 + w^2v^2 + 2(w + b)^2wv - 2(w + b)w^2v \]
\[ = (w + b)^4 + w^2v^2 + 2wvb(w + b) > 0 \]

QED Claim 5

2.D. \( 0 < \frac{\partial \Delta f_{S_i}(t)}{\partial f_{S_i}} < 1 \).

The increase in the precision of the investors’ beliefs due to the forecast of the analyst is:

\[ \Delta f_{S_i} = \frac{1}{Var(\pi_t|\pi^F)} - \frac{1}{Var(\pi_t)} = \left( 1 - \frac{1}{\sigma_{\pi_t}^2 \left( 1 - \frac{\left( \frac{s_{\pi_t}}{\sigma_{\pi_t}^2 + \sigma_{\pi_t}^2} \right) \sigma_{\pi_t}^2}{(1/2) \sigma_{\pi_t}^2 + \sigma_{\pi_t}^2} \right) \sigma_{\pi_t}^2 + \sigma_{\pi_t}^2} \right) - \frac{1}{\sigma_{\pi_t}^2} \]

In order that the proof of the common knowledge case will hold for this case as well, it is sufficient to show that when \( f_{S_i} \) increases by a marginal unit the increase in \( \Delta f_{S_i} \) will be less than that of a marginal unit.

Denoting \( \sigma_{\pi_t}^2 \equiv x, \sigma_{\pi_t}^2 \equiv y, v \equiv \frac{1}{y} \equiv f_{S_i}, \frac{1}{(2\beta)^2} \sigma_{\alpha_t}^2 \equiv z \), and taking the derivative of the above with respect to \( x \) we get:

\[ \frac{\partial \Delta f_{S_i}}{\partial f_{S_i}} = \frac{\partial \Delta f_{S_i}}{\partial v} = \frac{\partial}{\partial v} \left( x \left( 1 - \frac{\left( \frac{s_{\pi_t}}{\sigma_{\pi_t}^2 + \sigma_{\pi_t}^2} \right) \sigma_{\pi_t}^2}{(1/2) \sigma_{\pi_t}^2 + \sigma_{\pi_t}^2} \right) \sigma_{\pi_t}^2 + \sigma_{\pi_t}^2 \right) \right) \]
\[ = x^2v \left( \frac{2z+x^2v + 2z + x^2v}{(x^2v^2z + 2z xv + z + x^2v)^2} \right) \]
It is sufficient to show that

\[
x^2v \frac{2zxv + 2z + x^2v}{(x^2v^2z + 2zxv + z + x^2v)^2} < 1
\]

\[
x^2v (2zxv + 2z + x^2v) < (x^2v^2z + 2zxv + z + x^2v)^2
\]

which is equivalent to

\[
x^2v (2zxv + 2z + x^2v) - (x^2v^2z + 2zxv + z + x^2v)^2 < 0.
\]

Rewriting the expression yields

\[
-2zx^3v^2 - x^4v^4z^2 - 4x^3v^3z^2 - 6x^2v^2z^2 - 2x^4v^3z - 4x^2v^2z - x^2v^2 < 0
\]

which holds for all \(z, x, v > 0\). Hence

\[
0 < \frac{\partial \Delta f_{S_i}}{\partial f_{S_i}} < 1
\]

QED

**Appendix 3 – Proof of Corollary 2**

Assume that \(f_{S_i}\) increases by \(\Delta\), then the unconstrained optimal forecasting timing of analyst \(i\) decreases by \(\Delta\) (see Proposition 1). On the other hand, the “indifference interval” of analyst \(j\) which is of “size” \(f_{S_i}\) increases by \(\Delta\) as well. This in turn will decrease \(f_{L_i}\), but the decrease is by less than \(\Delta\). If before the increase of \(f_{S_i}\) analyst \(i\) was the first forecaster, then after the increase in \(f_{S_i}\) he will still forecast first at the time where the precision of the investors’ beliefs is \(\text{Min}\{ f(t_i^\ast), f_{L_i} \}\). Note that his forecast is in an earlier timing and lower precision of investors’ beliefs. If before the increase in \(f_{S_i}\) analyst \(i\) was the second forecaster, then following the increase in \(f_{S_i}\) both orders are possible. If analyst \(j\) still forecasts first, it will be at lower or equal precision of investors’ beliefs. In this case analyst \(i\) will either forecast immediately after analyst \(j\) where the precision of investors’ beliefs will be higher than before the increase in \(f_{S_i}\), or else he will forecast at his unconstrained optimum which is in lower precision of investors’ beliefs than before the change. If analyst \(i\) forecasts first, than it will be at lower precision of investors beliefs, and analyst \(j\) will forecast at higher precision of investors beliefs than before the change.